

A Maze of Dimensions

The base loop entire — fourteen steps from nothing to law · Working draft v008

21 August 2026

Contents

- What has changed since v007
- What changed in v007
- What changed in v006
- What changed in v005
- What changed in v004
- Part I — Definitions
 - Virtual
 - Existence
 - Relation
 - Leibniz's Law
 - Paradox, and the four states
 - Emergence
 - Dimension
 - Mandala
 - Two things I do not accept
- Part II — The transforms
 - Rotation, reflection, involution
 - Two reflections make a rotation
 - What the transforms do in the Mandala
- Part III — The shape of the loop
 - The governing rule
 - Why three reflections, and why four quadrants
 - The ordering, derived — and then confirmed twice
 - What each reflection is driven by
 - The loop of reflection, and the six pairs
 - What the powers of i actually do
 - The Mandala as guide
- Part IV — The fourteen steps
 - The Fire quadrant
 - The Earth quadrant
 - The Air quadrant
 - The Water quadrant
 - The closure

- Part V — What comes out of it
 - The Born rule
 - Time
 - The double cover, and whether the loop closes once or twice
 - What the closure hands upward
 - What the loop has produced, and what it has not
- Part VI — Positioning
 - The rival mathematical ontologies
 - Four objections a philosopher will make
 - Why zombies are not conceivable
- Part VII — Housekeeping
 - Changes made in v008
 - Changes made in v007
 - Corrections and changes made in v006
 - Corrections and changes made in v005
 - Corrections and changes made in v004
 - Where this goes beyond the record
 - What remains open
- Appendix A — the author's standing notes
 - A note on the meaning of inner and outer
 - A note on Paradox
 - On the nature of reflections
 - On the use of elements and signs
 - A directive on Tiers
 - The four quadrants
 - On the question of Reality
- Appendix B — parked reference notes

Working draft v008 — 21 August 2026. As v007, with Q3 and Q4 given their proper names, the author's note on what makes Reality real added to the standing appendix, the unit-sphere argument for the 4D hyperdox stated where it is used, and the tier above characterised as the story of behaviour rather than of being.

What has changed since v007

Four additions of the author's, one naming ruling, and one consequence that follows from putting the two together.

Q3 is Internality and Q4 is Individuality. v007 left the four kinds of Existence as Factuality, Reality, Individuality and Possibility, with *internality* entering only as a description of how Q3 and Q4 face. That was the cautious option and it is now overruled by the author, correctly. Q3 is the inward perspective itself and is best named for it. Q4 is what such a perspective leaves — a void defined in terms of a self, limited, standing apart, not factually alike to everything — and that is individuality in the strict sense. *Possibility* survives as what Q4's void consists in rather than as its name. Part I carries the author's note on why the choice of word matters when the things being named are invisible.

There is a dividend, and it was not designed. What the closure deposits into Fire at step 14 was called, in v007, the fact of individuality — a name arrived at from the deposit's own character, long before the quadrant was renamed. It now turns out that the quadrant handing the surplus upward is the quadrant the surplus is named after. Two roads, one junction.

What makes Reality real. A new standing note in Appendix A, and the most substantial addition since the Part I work at v006. It answers a question the document had been stepping around: why Euclidean objects stop being universal and become relative. The answer is that only perfectly 3D objects can be Euclidean, and anything foldable in four dimensions or more must be boosted in three. The unit sphere carries the argument: in 4D there is a novel region where radius and diameter shrink to half the 3D equivalent while the full 3D structure is retained four times over, so the radius takes more than one value depending on perspective. That is a hyperdox — multiple truths co-existing — and it is the one the first turn resolves. The argument is summarised where it is used, at Part II.

Folding against bending. The same note draws a distinction the document has been using without stating: extrinsic emergence folds a number and gives square objects, while intrinsic emergence bends it — rotates it about a point — and gives round ones. Both make area. The second is where negation comes from, since nothing about the differences between positive numbers forces negatives, whereas rotation through half a circle does. And the note's conclusion is worth the whole of it: the imaginary dimension is only the real dimension looked at from the other end.

Tier 2 is about behaviour. The note ends with a line that settles what kind of thing the tier above is. *If Tier 1 is about what things are, about the ontology of Existence in its various forms, Tier 2 is about the behaviour of Tier 1 — not what is, but what it does.*

That turns out to be the tier-boundary argument said backwards. Fire is the ledger of what things *are*, and what the closure sends up is a fact about what a particular whole was like from inside — a fact about a doing, not about a kind, and Fire has no column for it. So the surplus factuality cannot hold and the subject matter of the tier above are the same thing, and the boundary between the tiers is the boundary between being and behaving. Part V now says

so. With one qualification the note itself supplies: behaviour is not absent from Tier 1, since partition, addition, subtraction, multiplication, division and rotation are all things facts do. The difference is which of the two is the subject.

The note also observes that although Tier 2 has been assumed to equate entirely to Iconism, and one view has Iconism belonging to Q3, the concept may yet turn out to be applicable in all four quadrants. That is now the first question the interface note puts to Tier 2.

Smaller matters: the Mandala is described as a map of all possible combinations of relation, which fixes what can and cannot be criticised about it; and *unbounded is an adjective* is added where the verb-not-noun point is made.

What changed in v007

Four additions and one repair, and between them they close the largest structural hole the document had.

The ordering ruling now derives rather than merely confirms. v006 rested the ruling on two witnesses: my own Fire text, and the zodiac sign list. The second of those is the first stone anyone will throw — derived from astrology and confirmed by astrology. But the derivation has been sitting in Appendix A since v002, in a single clause, and it owes nothing to any chart. Emergence runs fire → earth → air → water; that is the direction in which paradox demands resolution, and it is fixed before any quadrant begins reflecting. A quadrant takes its targets in that direction, and cannot take itself, so it begins at its successor. Three targets, wheel order, successor first. That is now the argument in Part III, and the Fire text and the sign list are demoted to what they always were — independent confirmations of a result got some other way.

The loop of reflection is worked out, and it pairs all twelve sectors. Appendix A has long said that the first sector of one quadrant reflects in the final sector of the next, “creating a continuous loop of reflection that surrounds and is within the continuous loop of rotation.” Worked out against the ordering rule it is exact, it closes, and it goes further than the note claimed. Every sector has a mirror partner — *X-from-Y* against *Y-from-X* — and all twelve pair into six, with none left over and none doubled. Firsts pair with the next quadrant’s thirds; seconds pair with the opposite quadrant’s seconds. Part III has the table.

The reason it is worth having is not that it is new information. It is forced by the ordering rule, so strictly it is bookkeeping. It is that all six pairs *mean* something, and nothing made them: classical separation mirrored by entanglement, one allowance mirrored by every admissible future, and the first area ever made mirrored by the very area the Born rule squares. Those correspondences fell out of a rule fixed two versions ago for entirely different reasons. It is the strongest corroboration the ordering has, and the only one that owes nothing to the zodiac.

The closure deposits two things, not one. Step 14 said the base loop deposits law. It does, and it also deposits the fact that what closed was a *particular* whole. Those are the two halves of Leibniz’s Law and the document already has both: what is alike across instances becomes law, and what is unlike is still two, because the balls are in different places. The second half had gone unread, and it is the half that matters most, because it is what the tier above feeds on.

The tier boundary is derived. voo6 said that past the seven base dimensions the expansion goes fractal — said it, and did not show it. That was a hole of exactly the kind Part VI drills into other people’s theories: a forced foundation with an unforced superstructure. It is now closed. The closure delivers into Fire a fact whose content is that there was an inside, and Fire has no dimension in which from-here means anything. Actual, unpreventable, determinate on both sides. A new dimension is forced. **A tier begins where a closure deposits a fact that factuality has no room for** — which is step 1 again, run at the top of the loop instead of the bottom. Part V has the argument in short; tier_boundary_v002.md has it at length.

One word, separated. *Subjective* had been doing two jobs: inward-facing orientation, which is true of Q3 and Q4 at every tier including this one, and experiential, which is Iconism’s and is not claimed here. The first is now **internality**, and the definitions say so. This costs the qualia reply in Part VI its old formulation, and that section is rewritten accordingly — for the better, since the old version was quietly claiming that amplitude and the light cone were experiences, which is a fight this tier does not need and would lose.

Smaller matters: the *Sorites rather than Theseus* reading of persistence is stated in Part I, where it is the general form of what the nested triangles demonstrate; and the zombie argument is marked as needing to be run one storey up, with the reason given.

Everything else stands as it did. What follows is voo6’s own note, retained, and voo5’s and voo4’s beneath it.

What changed in voo6

Most of the work was in Part I, which had been the thinnest part of the document and is now the part that carries the most weight.

The heart of a whole is defined, and it earns its own subsection under *Virtual*. Every whole has a centre; a two-ended relation has one for free, since a midpoint costs nothing that the relation did not already contain. A triangle is the case that settles what “virtual” means, and it does it by way of an objection I had better meet than dodge: the centre cannot be reached by any terminating construction without importing a relation the figure does not contain. Quite so. But the midpoint route imports nothing, runs for ever, and never arrives — and every triangle in the sequence has the same centre. The parts may be replaced without end while the heart stays exactly where it was. That is what a virtual point is, and it is the same object as the centre of gravity the section opens with.

Higher dimensions break the picture, and the break is the best worked example in the document after $4 = 1 \times 4 = 2 \times 2$. I had assumed the midpoint construction scales, with the void shrinking faster as dimensions rise. It does not scale. In three dimensions the six edge midpoints of a tetrahedron are not the vertices of a smaller tetrahedron; they are the vertices of an octahedron, which is not a simplex and is not similar to anything in the figure, and it takes half the volume rather than an eighth. Add one dimension to a self-similar recursion and the recursion stops being self-similar. Emergence, watched happening, in a construction anyone can check with a pencil.

The set-theoretic objection is restated as a diagnosis rather than a rejection, and it is stronger for it. The construction needs the empty set to be many things while its own axiom of Extensionality says it is one thing — which is Leibniz’s Law biting the hand that

built the numbers. That is not a reason to throw the construction away. It is a statement of what the construction is missing, and paradox is what supplies it.

Benacerraf is added, as more than half an ally. His identification argument is a mainstream case against exactly the construction I reject, and his conclusion that numbers are not objects is close to mine that they are virtual facts. We part over tense, not over numbers. His second dilemma I do call falsidical, and the fallacy is locatable: it runs on a causal theory of knowledge, and there is no realm here for anything to travel from.

The count of erected dimensions is rewritten. The paragraph settling twelve against seven was correct and unreadable. Since seven is load-bearing at step 14 and in the closing summary, it needed saying in a way that can be followed.

Two errors of mine are corrected. The definition of *Relation* described two indiscernibles collapsing into one, which is the failure case and not a relation at all; it now says that the nodes have some facts in common and are two things as one virtual whole. And hyperdox was said to fall at the third position in Earth, when Earth has no earth-facing reflection at all — no quadrant reflects toward itself — so hyperdox is foundational there rather than positional. The old text contradicted its own next sentence.

Smaller matters: *Mandala* gets a definition in Part I instead of waiting for Appendix A; the claim against infinity is narrowed to completed infinity, which is Aristotle's distinction and all the theory needs; the rule that every new dimension implies a latent orthogonal is stated once in the definitions instead of being used four times without being declared; and the ordering paragraph in Part III spells out why the ruling matters to the Born rule, since in v005 it said so in nine words and nobody could be expected to follow them.

What changed in v005

One section, expanded. Part VI opened with a short piece on Tegmark's Level IV multiverse, which was the only rival the document named. It now surveys the field: Tegmark, Ontic Structural Realism, Wolfram's Ruliad, Smolin's relationalism, Wheeler's "It from Bit", and — as the friendliest neighbours rather than rivals — Causal Set Theory and Loop Quantum Gravity. Each gets Virtualism's position on it, and the section closes on the objection that all of them share and that Virtualism was built to answer.

Two things came out of writing it that were not obvious before. Ontic Structural Realism turns out to be the nearest rival rather than Tegmark, and the objection that is thought to sink it — relations cannot exist without relata — does not touch Virtualism, for a reason that is already in the foundations. And Causal Set Theory's slogan, "Order + Number = Geometry", is very nearly what the Fire quadrant does, which makes Sorkin's programme the closest thing in mainstream physics to numerogenesis and worth reading properly.

One new open question: LQG labels its spin networks with $SU(2)$ representations, which is the same machinery Part V wants for the double-cover calculation. Whether that is a coincidence or a lead is worth finding out.

What changed in v004

v003 did the hard structural work: it made one document out of two, wrote Fire and Earth out in full, fixed the within-quadrant ladder, and settled the step numbering at fourteen. That

spine survives here intact. What v003 could not do was digest the material I added to it, which arrived as three separate things — a set of definitions, a body of retrieved mathematics on the transforms, and a long narrative retelling of the whole loop written in one sitting. The result was that the loop got told twice: once informally under “Transforms and the Mandala”, and once again step by step. v004 tells it once.

Four substantive changes.

The ordering ruling is closed. v003 ruled that a quadrant’s three reflections take the non-self elements in wheel order beginning with the successor, and flagged the ruling as needing confirmation because everything downstream rests on it. The confirmation was sitting in `mandala_framework_v005.md` all along. The twelve signs are listed there as element-from-element, and they give the rule in full: earth-from-fire, air-from-fire, water-from-fire; air-from-earth, water-from-earth, fire-from-earth; water-from-air, fire-from-air, earth-from-air; fire-from-water, earth-from-water, air-from-water. Wheel order beginning with the successor, four times over, no exceptions. The Fire text and the sign list are independent witnesses. The question is answered.

The involution’s bookkeeping is ruled. It stands outside the count of four powers of i , for a reason given at step 1.

The transforms get a section of their own, and one result from it — that two reflections make a rotation, with the angle doubled — raises a question about whether the base loop closes at $+1$ or -1 in the double cover. That has a section too, marked as new and as mine.

The philosophical positioning is written out. The Google conversation put the theory in front of four standard objections and one rival ontology, and the replies belong in the document rather than in a chat log. Part VI.

Smaller matters: the $4 = 1 \times 4 = 2 \times 2$ paradox is restored at step 3, where it is the cleanest worked example the theory has and where v003 had lost it. Time gets a short section, because the conversation supplied the principle that answers a question Fire had left hanging. The retrieved mathematics that does no work in the argument is parked in Appendix B rather than dropped, so the threads worth pulling stay findable.

My standing notes are carried unaltered as Appendix A. They are apparatus, not draft, and nothing above supersedes them.

Part I — Definitions

In Virtualism all things are composed of emergent virtual relationships. Brave words, and they need explaining if anyone is to follow what comes after. Some of the definitions are contentious. Their justification mostly has to wait, or this document would never get started.

Virtual

Something is virtual if it is *as if* it exists. A centre of gravity is a thing, but a virtual thing, existing on an as-if basis. So is a number — the number two, not the numeral. It exists where there are two apples, and it is in neither apple.

Virtual is the umbrella term for everything in the ontology. Real things are virtual, ideal things are virtual, internal things are virtual. The word marks the substrate character they share, against the folk distinction between real things and abstractions.

The heart of a whole

Virtual can be taken as the sum of the parts — the whole that is no more than the parts. That is not all of it. Virtual can equally be the parts *of* a whole, where the parts together define a void, and where there exists an intrinsic whole bordering that void.

Every whole is also a heart. The heart may resolve to a precise virtual point, or it may define a void that the whole's own parts never reach. Both cases are instructive, and the second is the one that shows what the word *virtual* is for.

The simplest case settles the principle and costs nothing to establish. A relation is two-ended, and a two-ended thing has a midpoint. The midpoint is neither of the ends. It is a virtual node produced by the relation and belonging to neither term, and it arrives with the relation rather than being added to it. Call this the heart operation. Every relation carries one, and applying it imports nothing that was not already there.

Now take three nodes. There are three relations between them, the figure plainly has a centre, and the question is what it costs to get to it.

The objection to be met is that the centre cannot be reached at all except by importing a construction the figure does not contain — a perpendicular to each edge, say, or a line from each vertex to the opposite side. That objection is half right, and the half it gets right is the important half. Every construction that *terminates* at the centre has to add a relation that was not among the original three. Perpendiculars import orthogonality, which is a fact about right angles and not a fact about this triangle. Medians import a relation between a vertex and a virtual node. Both work; neither is free. (For precision, since the three are often run together: perpendicular bisectors of the edges meet at the circumcentre, altitudes at the orthocentre, medians at the centroid. Which centre one is after does not affect the point, because all three constructions import something.)

But there is a route that costs nothing, and it is the heart operation and nothing else. Each of the three edges is a relation; each therefore has its own heart. Take all three, and they are the vertices of a new triangle nested inside the first — the medial triangle, its sides half the length of their parents and parallel to them, its angles identical. Nothing has been imported. Only the operation a relation already carries, applied three times over.

And it can be applied again, and again, without end. The triangles nest, each half the linear size of the last, and the process has no terminus. It never arrives at the centre.

What it does instead is worth more than arriving would have been. **Every triangle in that sequence has the same centre.** The parts halve, and halve again, and the heart does not move. The parts may be replaced entirely, as often as you like and for as long as you like, and the whole's one intrinsic point stays exactly where it was. That is as clean a demonstration as this theory has that a whole is not the sum of its parts: here is a whole whose parts can be swapped out for ever while the thing they define is untouched.

So the heart is a fact. It is determined, it is invariant under the replacement of everything that determines it, and it is reachable in finitely many steps by anyone willing to add a relation.

And it is never occupied. No vertex is ever at it, however far the nesting runs. It is a point defined entirely by what borders it and inhabited by nothing — which is the description of a void, and which is the same object the Water quadrant meets again at step 11 as a hole with a shape. The centre of gravity that opened this section is that object seen from the other end.

Sorites rather than Theseus. The nesting is also the answer to a question about persistence that is usually asked the wrong way round. Theseus asks whether the ship is the same ship once every plank has been replaced, and the question only has force if identity is carried by the planks. On this account it never was. The fact is not made of its truth-makers; the truth-makers may be recycled without end while the fact stands, exactly as the heart stands while the triangles halve away beneath it. What is left over is a Sorites question — at what point does the bordering become too sparse to define anything — and a Sorites question is a hypodox, a matter of where a boundary has not been drawn rather than of which of two things a thing is. That difference matters later, at every point where the past is said to be preserved in its facticity though its realities have been recycled.

On the nature of the process. The unending nesting is of a kind with the ratios that extend into the voids formed between integers. In both, a whole is given at a stroke while the partitioning that would locate it in terms of parts never finishes. It is a verb and not a noun; unbounded and not infinite. The comparison should not be pressed as far as irrationality — the centre of a triangle is the average of its vertices and is perfectly rational, where $\sqrt{2}$ and π are not, and a mathematician will say so at once. What the cases share is the unboundedness of the approach and not the arithmetic of the destination. Zeno's dichotomy is the closer relative, and the halving here is literally his.

What happens in higher dimensions

The triangle's higher-dimensional relatives are the simplices — tetrahedron in three dimensions, 5-cell in four — and it would be convenient if the same picture simply scaled, with the void shrinking faster as directions are added. It does not scale, and the failure is worth considerably more than the convenience would have been.

In two dimensions the construction is perfectly self-similar. The medial triangle takes a quarter of the area and the three corner triangles take a quarter each; all four are copies of the parent, and nothing new appears.

In three dimensions it breaks. A tetrahedron has six edges, and their six midpoints are not the vertices of a smaller tetrahedron. They are the vertices of an **octahedron** — eight faces where the parent had four, not a simplex at all, and similar to nothing in the figure. The four corner tetrahedra take an eighth of the volume each, a half between them, and the octahedron takes the other half. So the central region does not shrink as the dimension rises. It grows, and it changes kind.

That is the result to keep, and it says what the theory says everywhere else. What a dimension does depends entirely on what it emerges from. Add one direction to a self-similar recursion and the recursion stops being self-similar: an object appears in the middle that is neither the parent nor a copy of it, and nothing available in the two-dimensional case predicts it. This is emergence watched happening, in a construction that needs no physics and can be checked with a pencil. What appears in the middle of a 5-cell I do not know, and it is on the open list.

The void of inner space that these figures border is developed at Quadrant 3, where the internality of a whole is what remains once its external relations have been subtracted. How a mind's ignorance stands to that void, and whether adding facts closes it or merely narrows it, is Iconism's question rather than this tier's. The connection is noted and left there.

Existence

Existence, with a capital E, is the totality of everything that is. It comes in four kinds of virtual emergent relationship: **Factuality**, objective wholeness; **Reality**, objective separation; **Internality**, internal separation; **Individuality**, internal wholeness.

A note on the choice of words: Words are pointers to things, labels attached to objects, not the objects themselves. This makes choice of word an art in itself, and when choosing words to attach to the invisible and the immeasurable, it can be tricky to pick the precisely perfect word. When labelling Q3 and Q4, that together form subjectivity, the words are necessarily open to debate. What is clear is that the ontological status of Q3 is hypodox, while that of Q4 is anodox. Q3 can be just as accurately labelled Internality. Q4 can be just as accurately labelled Individuality. To be individual is both to stand apart, absent external relations, and to not be factually alike to everything, i.e. to be limited, to define a void in terms of a self. Ultimately individuality is internal, and emptiness is possibility, although emptiness from a subjective perspective may find a void that is already tenanted by competition. Only an objective void is truly free, and even then only as far as it goes.

Possibility therefore survives as what Q4's void consists in rather than as its name. The quadrant is Individuality; what fills it, or rather what fails to, is possibility.

Separation is an emptiness: a void, a defined space where there is nothing beyond the external definition. That much is common to more than one of the four, and saying so at once prevents the obvious confusion, because it does not collapse them into one another. What differs is the fence. In Reality the void is bordered by real objects standing apart, and the bordering is objective. In Internality it is bordered by what a perspective has had subtracted from it. In Individuality it is bordered by the silhouette of what is not, which is the photographic negative the last step of the loop turns back into a fact. One emptiness; three fences.

Internality and Individuality, and a word that had been doing two jobs

Up to v006 the third and fourth kinds were called *subjective*, and that word was carrying two loads that need separating before either can be trusted.

The first load is **orientation**. A thing has internality when its relations face inward and it has no outward address. That is why the objects of the Air quadrant cannot be seen — not because they are hidden or small, but because there is nowhere to look — and why the objects of Water cannot be seen either, for the different reason that there is nothing there. Internality in this sense is a structural fact about which way a relation points. It is true of Q3 and Q4 in this loop and in every loop above it, and it carries no implication whatever about experience. The theory's oldest word for the condition is **idiotropic**, coined at step 1 for a being that turns inward and outward from nothing at the same time.

The second load is **experiential** — what it is like, the thing the Hard Problem is hard about. That belongs to Tier 2, and this tier does not claim it.

From here on, *internality* names the first and *subjective* is reserved for the second, or for places where both are meant and the ambiguity is deliberate. The distinction costs one word and buys a great deal, because without it the account appears to say that amplitude, interference, capacity and the light cone are experiences. It does not say that, has never needed to say it, and would lose the argument if it did.

Why the quadrants take these two names. The property of internality belongs to Q3 and Q4 alike, so naming one of them for it needs a reason. The reason is that Q3 is the pure case: it is the inward perspective itself, the view from inside with the outward address subtracted, and it is not yet anything more than that. Q4 is what such a perspective *leaves* — a void with a shape, limited, standing apart, and not factually alike to everything. That is individuality in the strict sense, and it is what the closure deposits into Fire at step 14. So the quadrant that hands the surplus upward is the quadrant the surplus is named after, which is tidier than the arrangement it replaces.

Two cautions on the naming, since it is easy to trip over. *Internality* with a small i is the orientation and belongs to both quadrants; *Internality* with a capital is Q3. And the word *subjective* is still correct of Q3 and Q4 taken together, where it means the inward side of the loop as against the outward — it is only the slide from there to *experiential* that is refused.

Relation

A relation is a whole thing formed by a virtual connection between two nodes. The connection is formed by the similarity between the nodes together with the difference between them. Without sameness there is no relation. Without difference there is nothing to discern, so the nodes have some facts in common, and are two things as one virtual whole.

(v005 had this as “the nodes have the same facts and are one thing under two names,” which describes two indiscernibles collapsing under Leibniz’s Law — the failure case, and not a relation at all. The reply to Wheeler in Part VI turns on a relation needing partial sameness *and* partial difference, and the old wording would have handed a referee the counter-example.)

Leibniz’s Law

Leibniz’s Law is a nickname for what gets translated as the Identity of Indiscernibles, or the Indiscernibility of Identicals. Writing to Clarke, Leibniz put it that “to suppose two things indiscernible, is to suppose the same thing under two names.” Max Black opposed him with a universe of two iron balls. I don’t accept that the balls can be identical, or that such a universe could exist; the balls are in different places, and spatial position is always discerning. Leibniz gets the credit for the rule I use, which is approximately his.

For working out what *virtual* means, take it this way. Any real object — any fermion — in any real universe must obey the Pauli exclusion principle, and so cannot be identical with any other real object. Virtual objects are abstractions, and they are identical on the one-size-fits-all principle: every such object with the same properties is effectively the same object, and will behave like every other such object, all else being equal.

Two consequences follow, and they matter later. Changing the virtual object changes every partaking object, by downward causation. Changing just one partaking object may instead

cause the virtual object to diverge, so that it becomes two. Gravity and momentum are where this gets its workout.

The rule has two halves and the document has tended to use only one of them. The half about sameness is what makes law possible: all electrons are one electron in the respect in which they are indiscernible, so all electrons behave alike, so there are laws. The half about difference is what makes individuals possible: Black's balls are still two, because they are in different places, and nothing about their likeness dissolves that. **Both halves come down together at the closure**, and the second is the one the tier above feeds on. Step 14 says so; §*What the closure hands upward* in Part V says why it matters.

It is also the glue between quadrants. Each quadrant supervenes on each of the others by virtue of being identical with them in some shared fact — that is what allows a fact produced under one modality to be recognised under another, and it is what makes the transition between tiers a matter of careful analysis rather than of a single leap.

The most universal virtual object is number, which is a variation on being and unity, produced and changed by paradox partitioning Existence into the number line. Every flat number so produced is whole, like every other such number: both a single number and its own unique partition of the whole. Each is an eternal fact that, once in Existence, cannot be done away with. The totality of virtual facts about such numbers overlay one another, forming roots as ratios that extend into the voids formed between integers, and that is what lets natural arithmetic function.

Read as downward causation, Leibniz's Law says that each new number formed adds a new possibility to every extant number, growing what it means to be a number. Materialists and logical positivists will object. My answer is that without this claim there can be no material thing, and no logic either.

Where the set-theoretic construction goes wrong, and where it does not

Peano's axioms, von Neumann's ordinals, Dedekind's cuts and Cauchy's sequences build numbers in a fashion not far from the one described here. The differences are subtle enough to be worth stating exactly rather than waving at, and the way to state them is to let set theory summarise itself first:

A **natural number** is a nested set of empty sets. An **integer** is a set of pairs of natural numbers. A **rational number** is a set of pairs of integers. A **real number** is an infinite set of rational numbers — either a Dedekind cut or an equivalence class of Cauchy sequences. Therefore a real number like π or $\sqrt{2}$ is ultimately just a massive, beautifully structured collection of empty sets.

Three responses, in increasing order of weight.

First, “infinite” is doing work it cannot do. *Boundless* would be better and *unbounded* is the word this document settles on. A real number is not a completed collection of anything. The distinction is developed under *Two things I do not accept* below.

Second — and this is the one with teeth — the construction needs the empty set to be many things, and its own axioms say it is one thing. Extensionality, in Zermelo-Fraenkel, states that sets with the same members are the same set. There is therefore exactly one empty set, and every occurrence of $\emptyset$ anywhere is that same object. Yet

Zermelo's 3 is $\{\{\{\emptyset\}\}\}$, in which one object has to occur at three depths and be told apart by where it sits; and von Neumann's 3 is $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, in which it occurs four times over. The individuation is being done by structure, and the thing individuated is by the theory's own axiom a single thing. That is Leibniz's Law biting the hand that built the numbers: indiscernibles that the construction requires to be several.

I do not take this to show the construction is worthless. I take it to show precisely what the construction is missing, which is a better thing to be able to say. Set theory is right that you begin from the void, and right that unity has to come from somewhere. What it never says is where. It helps itself to unity and to a plurality of the void in the same breath, and calls the helping a definition. Paradox supplies both. The void turned inside out by the impossibility of being nothing and yet being complete is unity; and the same paradox partitions that void so that every version of it is at once identical with every other version, being indiscernible, and individually different from every other version, being its own partition of the whole. That is not an embarrassment to be tidied away. It is the engine.

Third, a concession, because it costs nothing and it is true. Von Neumann's ordinal n is the set of every number before it — each number the accumulation of every partition so far, which is remarkably close to what is claimed here and closer than Zermelo's pure nesting. And Peano's successor function is, step for step, what unity reflecting against the most recent partition does. The dispute is not about the arithmetic. It is about whether the recursion is a definition or an event. Peano writes down a rule; on this account the rule is what a forced sequence looks like once it has run, and the forcing is the resolution of the hyperdox of being against the last partition made. We agree about the numbers and disagree about whether they were ever built.

The bottom line. Numbers are still being created, and the rate of creation cannot outrun the pace of the integers. Ratios and roots are partitions of what the integers have already produced; they are downstream of the integer count and can never get ahead of it. Achilles cannot beat the tortoise because the track is unfinished — the game is rigged in favour of the tortoise, who starts at the finishing line and lays the track as he goes. This is the same claim made at step 2, that the final integer partition is always one more than the quantity of integers in Existence, and it is why the cutting edge of number sits at the opposite end of the spectrum from Cantor's diagonal argument.

Benacerraf, who is more than half an ally

Paul Benacerraf put two problems to the philosophy of mathematics. Virtualism has a different answer to each, and the first answer is mostly agreement.

The first problem, from *What Numbers Could Not Be* (1965), is the identification problem. Zermelo's 3 and von Neumann's 3 are different sets, nothing in arithmetic decides between them, and any identification of numbers with particular sets is therefore arbitrary. Benacerraf concludes that numbers cannot be sets — and goes further, that numbers are not objects of any kind, but positions in a structure.

On the first half I agree entirely, and it is worth saying so plainly, because it is a mainstream argument against exactly the construction I reject and it is not mine. On the second half I agree further than might be expected. A number is not an object. It is a virtual fact: the fact of a whole partitioned in a particular way. There is a property of threeness, which is the fact of a whole partitioned into three parts, and it emerged as a truth immediately after twoness

emerged and by the same process. Any whole that does in fact consist of three things — points, sides, angles, apples, stars — necessarily possesses the same virtual fact of threeness that originally emerged. The intrinsic void of individuality is bordered by threeness in the extrinsic reality of three stars, and the same fact is instanced again in whatever a mind does in response to three sources of photons.

Where I part from Benacerraf, and from the structuralism that descends from him, is over tense. He leaves the structure standing there, unexplained and unproduced, which is the same failing charged against Tegmark and Ontic Structural Realism in Part VI. Structure on this account is not found. It is forced, in order, one partition at a time.

The second problem, from *Mathematical Truth* (1973), is the dilemma. Either mathematical statements have the same semantics as ordinary ones, in which case they are about abstract objects and it is a mystery how a physical brain could come to know anything about them; or knowledge is kept human-scale, in which case mathematics is a language game and its unreasonable effectiveness in predicting planetary orbits becomes the mystery instead.

This one I do call falsidical, in Quine's sense, and the fallacy is locatable rather than a matter of taste. The dilemma runs on a causal theory of knowledge — that to know about X you must stand in some causal traffic with X, and that abstracta, being outside space and time, can supply none. Deny that premise and the dilemma has no horns. On this account there is no separate realm from which anything would have to travel. The same virtual fact is instanced wherever a whole is partitioned that way, and instancing is not transmission. There is one fact with more than one instance, which is what a virtual object is.

How far that answer goes is not settled here, and it should not be overclaimed. What Virtualism is committed to is that a mental instance of a fact and a physical instance of it are instances of the same fact. Whether the mental instance is *identical* with the physical one, and what it takes for a mind to be in possession of a fact rather than merely correlated with it, is Iconism's question. It is on the open list in Part VII rather than answered here. I have used Sherlock Holmes as the example in the past and he is a poor one: "in possession of the facts" is the very thing in dispute, so the detective assumes what he was hired to find out.

Paradox, and the four states

A paradox is a situation that appears logically impossible. Quine set the veridical and the falsidical aside as non-paradox and called what remained antinomy. Those are puzzles of logic. When it comes to Nature — what is, rather than what is merely claimed — it seems Nature abhors a paradox more than a vacuum. The greater the impossibility, the stronger the creative force that brings about emergence. The full story of emergence, from nothing through to spirit, is the tale of Nature avoiding contradiction.

I take the paradoxical states to be descriptive of ontological states, and to be the natures of the only four possible relational types. Orthodox and hypodox are borrowed, the latter from Peter Eldridge-Smith. Hyperdox and anodox I have coined.

- **Orthodox** — fully determined. Classically true. Sameness. The eternal past.
- **Hyperdox** — over-determined. Determinate on both sides, actual rather than descriptive, and unpreventable. Difference. The present.

- **Hypodox** — under-determined. Consistent but ungrounded; settled in some respects and open in others. Separation and inner change. The imminent future.
- **Anodox** — non-determined. A defined void: not nothing, but nothing settled with some boundaries. Internal wholeness; a void defined in terms of a self. The distant future.

Note that we are not considering propositions and language here but factuality and reality, so veridical, falsidical and antinomy do not apply. Dialetheia abounds in all four states one way or another.

On the self-referential paradoxes I am unsympathetic. The assumption in all of them is that something can be self-referential in more than the hand-waving way of words. Grelling's paradox is the perfect example. Take its words as words rather than as the facts the words point at, or perhaps only wave at, and you see that words can tell lies, and that words can never *be* the truth — at best they tell it. "Short" is not short compared with "long", which is shorter. "Syllable" is in no wise like a syllable, yet we are asked to believe that "multi-syllabic" is like many syllables. Grelling's argument is in tatters, and it deserves to be.

Emergence

Emergence is the coming into being of novel existence. It happens two ways: by the probable rearrangement of some under-determined situation of hypodox, populating a region of non-determined anodox; or by the necessary resolution of over-determined hyperdodox, forcing the creation of a novel dimension. The first is weak, the second strong.

Two predecessors are worth naming, because both built their philosophies on the premise that reality is a process driven forward by internal contradiction.

Hegel. Everything that exists contains its own contradiction, and the tension forces it to become something else. His *Science of Logic* starts where I start: try to think Pure Being — existence with no qualities — and it is completely empty, so Pure Being is identical with Nothingness. The mind cannot hold Being = Nothing, and the resolution is Becoming, reality as an ongoing process. The end of the unfolding, for Hegel, is the universe achieving self-awareness: abstract logic hardens into nature, then wakes as Subjective Spirit.

Laozi. Chapter 42 of the *Dao De Jing* gives a generative sequence from nothing to everything: "The Dao produces One. One produces Two. Two produces Three. Three produces all things." Where Western thinkers avoided the void, Laozi made it the source of all utility — the hole in the hub is what makes the wheel useful, the emptiness in the pot is what makes it a pot. And reality for him is a shifting dance of opposites in which the paradox is the harmony.

Where I part company with both is over number. Hegel thought mathematics too rigid and cold to be the fabric of reality, and put logic and concepts at the heart of things instead. Laozi treated number as a byproduct of the Dao and attended to flow rather than precision. Putting number at the heart of a paradoxical unfolding marries Pythagoras to Hegel, and makes the contradictions that drive reality forward *mathematical and structural* ones. That is the detour, and it is the whole difference.

Hegel also has the engine external and I have it internal. He needed the absolute to externalise itself into nature before it could get any generative friction. On my account the

friction is in the uncompletable resolution of 0 and 1, which is the cause of 2, and of 3, and of 4, and of everything after; and all of it has to emerge within that same scope of non-being and being, where being is simply the virtual wholeness of non-being.

Dimension

A dimension is a direction, defined as orthogonal to other dimensions, and populated by relationships. A direction can be called a vector, so orthogonal vectors are dimensions.

Dimensions are not a backdrop that nodes sit in. They are produced by the differences between nodes. Before there is a relationship of difference there is no dimension to occupy; after, there is.

Every dimension implies its orthogonal, and a new one implies a latent orthogonal. Any relation emerges from a pairing of nodes: the similarities bond them, the differences keep the two virtual wholes from collapsing into a single identity. Those differences populate one or more dimensions. And every dimension of difference, novel or not, implies its orthogonal. Where the dimension of difference is itself new, the orthogonal it implies has nothing in it — no second difference has yet arisen to run along it. It is latent: a direction that exists as permission and not yet as fact.

That rule is used four times in what follows and it is better declared once here than assumed four times. The point defines a possible line away from it. The line defines a possible second direction. An allowance defines a possible second allowance. In each case the new direction is real as permission before anything populates it — and the Born rule argument in Part V turns on exactly this, since an allowance with nothing standing against it is one end of a relation and therefore not yet a fact.

What a dimension does, how it affects behaviour, is entirely dependent on what properties it emerges from. We see that while a series of reflections initially gives extrinsic dimensions, eventually the novel dimension is forced to turn inward as the intrinsic fourth dimension of boost. This turn inward is the overriding fact that places reality inside of factuality, and it leads to a second inward turn, then a third and a fourth, creating four supervening regions carrying twelve extrinsic dimensions between them. The emergence does not stop there but continues, fractal-like, forming a complete Existence that is mapped by the two-dimensional image that is the Mandala.

Mandala

Because the Mandala is formed by four emergent states, the pattern that emerges with those states — at least in the early stages — looks very much like the zodiac of astrology. Because of that, and because the sectors need labels, I name the sectors formed by each state fire, earth, air and water, after the classical elements, in much the same way that QCD names colour charge as red, green and blue.

None of this requires a commitment to astrology, though I will admit that astrology was my inspiration for the whole of Virtualism. Chemistry was built on the ruins of alchemy and astronomy on the remains of astrology, and acknowledging where a discipline came from has never yet made it wrong. The physicist borrows red, green and blue from the eye to label a triple charge that has nothing whatever to do with light. I borrow the zodiac to label a four-

way and twelve-way structure that has nothing whatever to do with horoscopes. Duchamp would recognise the manoeuvre.

It is worth being clear about what the zodiac is doing in the argument and what it is not, since voo7 has changed the answer. It is not a premise. The ordering rule that everything downstream depends on is *derived* in Part III from the direction of paradox, and would stand if no chart had ever been drawn. The sign list is a check that the derivation had to pass, and it passed. A test that could have failed is evidence; a source that supplies the answer is not, and this is the first.

Two things I do not accept

Completed infinity. Infinite regress is impossible, because any such tale would still be stuck on the prologue. The story must have a beginning, and the beginning of everything must be a beginning of a kind that needs nothing before it. More generally, an actual or completed infinity is a category mistake: *infinite* is an adjective meaning unbounded, unlimited, endless, and an endlessness that has been finished is not an endlessness. The distinction is Aristotle's, between the potential infinite and the actual, and I am on his side of it.

What I am not doing is banning unbounded processes. The theory could not run without them. The number line is unbounded; the ratios fill every interval without exhausting it; the nesting of medial triangles never terminates. All of those are verbs. None of them is a completed quantity, and *unbounded* is the word that keeps the distinction visible, because it shifts the claim from quantity to potential, which is where it belongs.

The set-theoretic construction of number. The full objection is made under Leibniz's Law above: nesting empty sets — $\{\}$, $\{\{\}\}$, $\{\{\{\}\}\}$ — as Zermelo did, or accumulating them as von Neumann did, helps itself both to unity and to a plurality of the void without accounting for either, while the theory's own axiom of Extensionality says the void is one thing. What is left once that is granted is the deeper objection, and it is again about tense. The construction is static. It produces a library of numbers rather than a production line. On my account integers are churned out as a sequence of anodox and orthodox resolution, and that sequence is unbounded. The process is unstoppable. It is a verb, not a noun; unbounded is an adjective.

Part II — The transforms

The Mandala is a two-dimensional picture of something that starts simply and develops into a collection of hyper-dimensional spaces and objects. Every step that produces novelty does so through creation or rearrangement. Seen more clearly the two are one thing, causation, but the differences are real enough to reveal four states with four steps between them: emergence, subtraction, demergence, addition. For the simple cases those steps are describable mathematically, as combinations of rotations and reflections. So the mathematics is worth stating plainly before it gets used.

The Mandala is a map of all possible combinations of relation, therefore it provides a visual aid to comprehending everything that can exist — that is, the Mandala allows the explorer an abstract map of Existence. Because of its mathematical nature, the only criticism that can be

levelled must be about the fundamental nature of virtual relations. Beyond that, everything is a matter of interpreting the meaning of the map.

Rotation, reflection, involution

All three are linear maps on a vector space.

A **rotation** preserves the inner product and has determinant $+1$. It is rigid motion with handedness left alone. In n dimensions rotations do not happen about a point or an axis, except by accident in two and three dimensions; they happen within orthogonal two-dimensional planes, and an n -dimensional space admits $\lfloor n/2 \rfloor$ of them at once. The group is $SO(n)$.

A **reflection** fixes a hyperplane of dimension $n-1$ pointwise and inverts the direction orthogonal to it. Determinant -1 , so handedness flips. Three-dimensional space reflects across a plane, four-dimensional space across a three-dimensional hyperplane.

An **involution** is any map that is its own inverse: $f(f(x)) = x$. Every reflection is an involution, since reflecting twice returns you to where you were. Central inversion $-x \mapsto -x$ — is an involution in any dimension, and in even dimensions it is orientation-preserving, which makes it a proper rotation rather than a reflection.

That last point is worth holding on to. Whether a given inside-out turn counts as a rotation or a reflection depends on how many dimensions it has to work with. The Mandala's turns behave differently at different depths for exactly this reason.

Two reflections make a rotation

Compose two reflections across hyperplanes meeting at angle θ and you get a rotation through 2θ . In Clifford algebra the reflection of a vector v in the hyperplane normal to a unit vector u is the sandwich $-uvu^{-1}$; do it again in the hyperplane normal to w and the composite is $(wu)v(wu)^{-1}$. Write $R = wu$. Since u and w are unit vectors, R is a rotor, which is to say a spinor, and it rotates v by 2θ in the plane spanned by u and w .

The doubling has a consequence that is usually presented as a curiosity and is not one. To rotate vectors through a full 360° , the angle between the reflecting hyperplanes must be 180° , so $w = -u$ and $R = (-u)u = -1$. Apply that to an ordinary vector and the minus signs cancel: the vector is exactly where it started. But the rotor has gone from $+1$ to -1 . Getting the rotor back to $+1$ needs $\theta = 360^\circ$, which is a 720° rotation of the vectors.

Vector rotation (2θ)	0°	360°	720°
Spinor state (R)	$+1$	-1	$+1$

In group terms: rotations in n dimensions form $SO(n)$, and pairs of unit reflections form $Spin(n)$, which double-covers it. For every rotation matrix there are two spin elements, $+R$ and $-R$. A spinor is an object that remembers which reflections composed the rotation, rather than only the net motion of the vectors.

What the transforms do in the Mandala

What is seen initially is a reflection of nothing and something, where either is the negation of the other. Since we are positing a beginning to avoid infinite regress, it is the negation of nothing that is something.

That state of being I call fire, virtual fact. It is paradoxical in itself: classically true, and in being orthodox truth also over-determined hyperdox, because the double node of the relationship of being is also two, having partitioned everything that exists. This is perhaps what Laozi was recorded as claiming.

The hyperdox-orthodox sequence produces the first extrinsic dimension, the number line. It is within Existence as a whole, but I call it extrinsic because nothing contests it. It forms a straight line running from 0 through 1, 2, 3 and so on, and each integer is a unified whole in itself. Therein lies the paradox that lets numbers function arithmetically. That kind of emergent form I label earth, a state of differences.

If the number line is produced by reflection, the ratios are produced by superposition — an intrinsic many-worlds of possibility and probability.

The line itself defines the next dimension as orthogonal, even before anything populates it. At that stage the second dimension is anodox: a non-being that is somehow there as possibility. Extending the whole number line along it is made probable by the ratios, which when turned inside out form area by multiplication replacing division. The linear number that forms an area has been folded and massaged into being something ontologically other than a line — still the same number, but with a different shape, 2D rather than 1D. Probability lets this happen as weak emergence. It happens because it can, and being another extrinsic dimension it is still unopposed. That form I label air, a state of 2D being that allows complex number, and from complex number we get rotation about a point, circles, negation about the point, addition, and then reflection as negation across a line, all in 2D.

The same process runs again with a second reflection, this time of 2D area, giving 3D volume, also unopposed. More number origami. That form I label water.

The story so far is of being naturally forming a set of mathematical forms simply by increasing the quantity of integers — which is the only strongly emergent dimension yet.

In emerging as 3D with rotations and reflections, the numbers form what we can only sensibly call an ideal space; sensibly, because to call it anything else would be perverse. This arena is mathematical and geometrical, a realm of truths and facts that are all the same, which confers on them inevitable relationships of fact belonging to Euclidean space. A limited Platonic space, if you like, but not a Meinong space. It is not the case that anything goes.

What comes next is not obvious at first. Within the mathematics of rotation there are Pythagorean right angles, and these define precisely what complex numbers are as true objects, and from that we get spheres and balls from numbers, also as true objects. Meanwhile the reflection of lines and planes defines spaces that are not necessarily occupied. Those spaces correspond to fields, and can for instance support the ideals of holograms as wholes with external parts. Populating such space with factual objects is another matter, or

rather non-matter. The space itself is anadox and its under-determined status is hypodox, and together they allow rearrangements — rotations and reflections.

Once the origami of number reaches 4D a difficult paradox appears. The folding may occur because it can, as weakly emergent extrinsic rearrangement of the facts, but in doing so a hyperdix arrives: the differences between centres of facts as 4D no longer match the differences between the same centres as 3D. The 4D version is factually smaller in radius than the 3D version.

This point seems trivial, but to stem objection I would stress the nature of the unit sphere, where in 3D the radius 1 equates to $x^2 + y^2 + z^2$, while in 4D we see $1 = w^2 + x^2 + y^2 + z^2$. The 4D unit sphere differs in that it has a novel region where the radius and diameter shrink to $\frac{1}{2}$ that of the 3D equivalent, while retaining the full 3D structure four times over. The virtual argument is that this fact produces an apparently paradoxical situation whereby there can exist multiple values for the radius, depending on the relative perspective. This is what is meant by a state of hyperdix. The matter is taken further in Appendix A, under *On the question of Reality*.

The nature of facts is that they are true of themselves and remain so, and as with number that property gives them generative power when they meet paradox. With 4D objects we get the unavoidable introduction of boost, a rotation into what we call imaginary space — though the space is no more imaginary than ideal Euclidean space. It is space reached by rotation, using the same simple facts that gave us 2D circles and Pythagorean triangles.

Boost applies to every perspective of the 3D object. So while we can reach it with one imaginary rotation, it is equally necessary that the entire 3D space be reflected in the process. The rotation is also a reflection, and that composite has a name: involution. Involution turns a space inside out, and it is the transform that forms the boundary between Quadrant 1, the fire quadrant of factuality, and Quadrant 2, the earth quadrant of reality. There was already a prototype for it at the boundary crossed from non-being to being, where the void was reflected as the fact of unity by addition.

All of this happens because it must. I do not see any way for there to be another version of Existence — an Existence MkII forming another universe from another void with another set of numbers. The actual Universe, with its history of facts, ought to be plenty for anyone, and while it may do a great many things I cannot see that it can admit an entirely separate collection of facts. So the rule is that all emergent novelty has to be based on the emerging number line as the fundamental facts of Existence. The story is one of Existence bootstrapping itself, and it requires no creator.

If anyone has been watching me develop this and wondering why I keep going over the same ground, the answer is that I am trying to be crystal clear about the next step, because that step is not measurable in the way reality is. We have to deduce and infer the reasons for what we already know to be the case but have no explanation for. That means the foundations of consciousness, and the Hard Problem.

Part III — The shape of the loop

The governing rule

One rule makes the rest legible, and it belongs before the steps rather than inside them.

There are two directions of emergence and they co-exist. Within an element the flow runs **outward** — linear, unrestricted, driven by what is under-determined, populating a dimension that is already open. Between elements, where the over-determination strikes, the flow runs **inward** — rotational, capped at ninety degrees, erecting a dimension that was not there before. Three outward reflections, then one inward turn; three again, then one; four times over, until the algebra of i brings us home.

The outward reflections are the extrinsic dimensions and the inward turns are the intrinsic ones. Twelve of the one, four of the other.

And that settles the counts that have been running side by side and confusing matters. Sixteen dimensions arrive in all: twelve extrinsic, three to a quadrant, and four intrinsic, one to a turn. But not all sixteen are new *directions*. Only seven had to be built where nothing was available before — the three Euclidean directions of Fire, which had only a point behind them, and the four turns, each of which forces open an orthogonal that was not there. The other nine reflect inside room that the turn before them had already opened. So sixteen counts dimensions; twelve and four count how they arrived; seven counts what was erected. They answer different questions and all three are right.

Seven is the count that matters later. It is why the closing step at 4Di deposits a *seventh*-dimensional fact, and why the expansion past the base loop has to go fractal rather than carry on adding quadrants. If the paragraph seems like bookkeeping, that is what it is; but two later claims fall over without it.

Why three reflections, and why four quadrants

The numbers are not chosen; they follow.

Why three. A quadrant reflects toward the elements it is not. There are three of those, and it takes them in the wheel's order, beginning with its successor. Fire reflects toward earth, then air, then water. Earth toward air, then water, then fire. Air toward water, then fire, then earth. Water toward fire, then earth, then air. Three reflections exhaust the targets.

A fourth reflection would have to return to the successor, which the first reflection already took. That return is a duplication: the same relation asserted twice, with no new dimension to distinguish the second assertion from the first. Duplication of that kind is over-determination in the strict sense — determinate on both sides, actual rather than descriptive, and unpreventable, since no rearrangement within the quadrant will flatten it. That is hyperdox, and hyperdox cannot be held flat. The only move left is inward.

So three is not a preference for triads. It is how far a sector can be populated before it repeats itself, and repetition at that depth is paradox.

Why four. Four is the number of right angles in a full turn, which is the number of times you can multiply by i before you are back on the real axis. Three because the targets run out, four because the algebra closes, twelve because they multiply.

What the turn erects. This falls straight out of the ordering. The fourth reflection, the one that cannot be made, was reaching for the successor element. Denied the outward route it takes the inward one, and what it erects is that same successor, held intrinsically rather than reached for extrinsically.

- Fire's fourth would have reached for **earth**, which is difference. Turned inward, difference becomes **mass** — boost, the objective difference a thing carries in itself.
- Earth's fourth would have reached for **air**, which is separation and inner change. Turned inward, separation becomes **momentum** — a commitment to change relation, held with nowhere to point.
- Air's fourth would have reached for **water**, the defined void. Turned inward, the void becomes **un-boost** — the surrender of inner value, absence with a shape.
- Water's fourth would have reached for **fire**, which is fact. Turned inward and back onto the real axis, fact becomes **a thing becoming true**.

Four turns, four intrinsic dimensions, each the interior of what the outward route could no longer reach. It is the same structural fact seen from two sides: what the quadrant ran out of, and what the turn built.

The ordering, derived — and then confirmed twice

v003 ruled this and asked for confirmation. v004 supplied two witnesses and closed it. What neither did was notice that the ruling is not a ruling at all but a consequence, and that the derivation had been sitting in Appendix A since v002 in a single clause. Since everything downstream rests on the ordering, and since one of the two witnesses is the zodiac, the derivation should lead and the witnesses should follow.

The derivation, from the direction of paradox. The reflections are not free to take their targets in whatever order they please, because only one order is available to them. Emergence runs fire → earth → air → water. That is the causal order — the direction in which any inherent paradox demands its resolution within nature — and it is fixed before any quadrant begins reflecting, since it is what makes a quadrant follow a quadrant at all. A quadrant reflecting outward therefore takes its targets in that direction and no other. It cannot take itself, because a node reflected against itself is not a relationship and so not a fact. So it begins at the next element along, which is its successor, and continues in order until the targets are exhausted.

Three targets, in wheel order, beginning with the successor. That is the whole rule, and it follows from the direction of paradox plus the prohibition on self-reflection. It owes nothing to any chart, and it should have been in the body long before now.

First confirmation, from my own Fire text. The original names its own targets, written before the rule was stated. At 0D → 1D the number line emerges and I call it “the beginning of difference” — difference is earth. At 1D → 2D I say in terms, “for convenience we can label this as air from fire.” At 2D → 3D the volume arrives and I label it “water, as in the Platonic elements.” And the break at 3D → 4De goes into earth. So Fire's three reflections are earth, air, water, in wheel order beginning with the successor, and the break returns to the successor.

Second confirmation, from the zodiac. The twelve signs are listed in mandala_framework_v005.md §8 as element-from-element, and the list is the rule written out:

Quadrant	Signs	Reads as
Fire/red	Capricorn, Aquarius, Pisces	earth-from-fire, air-from-fire, water-from-fire
Earth/yellow	Libra, Scorpio, Sagittarius	air-from-earth, water-from-earth, fire-from-earth
Air/blue	Cancer, Leo, Virgo	water-from-air, fire-from-air, earth-from-air
Water/green	Aries, Taurus, Gemini	fire-from-water, earth-from-water, air-from-water

Every quadrant takes the three non-self elements in wheel order beginning with its successor, and without exception. Since the Mandala is reverse-engineered from the zodiac, and the whole point of that derivation is that the structural account has to recover the zodiac as its first-order elaboration, the agreement is exactly the test the ruling needed to pass. It is a test that could have failed, which is what makes it evidence; had the derivation from the direction of paradox given some other order, the sign list would have contradicted it and one of the two would have had to go.

So the table stands:

Quadrant	first reflection	second	third	turn into
Fire (orthodox)	earth from fire	air from fire	water from fire	earth
Earth (hyperdox)	air from earth	water from earth	fire from earth	air
Air (hypodox)	water from air	fire from air	earth from air	water
Water (anodox)	fire from water	earth from water	air from water	fire

Why the ruling matters more than a ruling should

Air's order is water, fire, earth, which is what v002 assumed — and the consequence is larger than a piece of bookkeeping has any business being, so it is worth spelling out rather than asserting in one line as v005 did.

Within a quadrant the three reflections give a line, then an area, then a volume. So Air's first reflection, toward water, is the **line** — the allowance, the bare extent of a possibility's claim on the whole, which physics calls the amplitude. Its second, toward fire, is the **area** — where two allowances agree or cancel, which physics calls interference. **The allowance is the line and the interference is the area, not the other way about.**

That is what the Born rule argument needs, and it needs it exactly this way round. A line is a single node with nothing standing against it, so it is not a relationship; on the foundations a fact is a relationship; so a line is not a fact, and only facts can be observed. The first observable thing in the quadrant is therefore the area. Reverse the two rungs and the allowance itself becomes the first fact, you would observe the amplitude directly, there would be nothing left to square, and the argument in Part V collapses entirely. The ordering is what keeps it standing.

The rival ordering voo2 worried about — fire, earth, water — is the wheel taken from its own beginning rather than from each quadrant's successor. Applied as a general rule it breaks at once: Fire's three would then be fire, earth, water, and Fire would be reflecting toward itself first. A node reflected against itself is not a relationship and so not a fact. That ordering was never available.

(That last argument and the modulus-square repair in Part V both turn on when a reflection against oneself does and does not make a relationship. They are compatible, and the reason is given in Part V under *the correction that improves the argument*: the complex conjugate is a genuinely distinct node, being a different position on the plane, where a node reflected against itself is the same node named twice. A reader who notices the tension here should look there before objecting.)

Ruling: closed, and now derived. Reflections within a quadrant take the three non-self elements in wheel order beginning with the successor; the turn returns to the successor and erects it intrinsically. Derived from the direction of paradox, and confirmed independently by the Fire text and by the sign list.

What each reflection is driven by

With the ordering fixed, the modality of each reflection follows, and it is the modality of its **target**, not its position.

The four relational types run in a cycle — orthodox, hyperdox, hypodox, anodox — and that cycle is the elemental order: fire is orthodox, earth hyperdox, air hypodox, water anodox. A reflection toward earth is hyperdox-driven wherever it occurs, a reflection toward air is hypodox-driven, and so on.

Read off the table, Fire's three run hyperdox, hypodox, anodox, which is what my note on reflections says: "the orthodoxy of numbers leads to 1D hyperdoxy... reflecting the line is 2D hypodoxy... hypodox reflects again because it can, defining the anodox of volume." Earth's run hypodox, anodox, orthodox. Air's run anodox, orthodox, hyperdox. Water's run orthodox, hyperdox, hypodox.

This is also what I meant by "hyperdox will occur at different relative positions within each quadrant." It does: first position in Fire, third in Air, second in Water — and **foundational in Earth**, which is the case that had been got wrong. Earth's three targets are air, water and fire; earth is not among them, because no quadrant reflects toward itself. So hyperdox occupies no extrinsic position in Earth at all. It is the quadrant's own modality rather than one of its rungs. (voo5 said "third in Earth", contradicting the sentence immediately above it. The pattern is simply the wheel offset, and it comes out right in every quadrant once that is seen.)

What does not move is the break, which is always the duplication after the third, and therefore always over-determined regardless of which target it duplicates.

voo2 had all four quadrants running a fixed ladder of under-determined, non-determined, over-determined. That was wrong twice: it froze what rotates, and it put non-determination between under- and over-determination when the settled architecture runs non-determined (distant future), under-determined (imminent future), over-determined (present), fully

determined (past). The correction costs the Born rule argument one of its two props and replaces it with a better one.

The loop of reflection, and the six pairs

New in v007. The claim is the author's and has been in Appendix A since v002; the working-out and what it turns out to corroborate are mine.

Appendix A records that the break into Earth “is both a reflection of the factuality of fire and a surrounding of the first quadrant, which sees the first sector of Q1 reflected in the final sector of Q2 and so on throughout the quadrants, creating a continuous loop of reflection that surrounds and is within the continuous loop of rotation.”

That has never been worked out. Worked out, it is exact, it closes, and it says more than the note claimed.

Take the mirror of a sector to be the sector with its terms exchanged: the mirror of *X-from-Y* is *Y-from-X*. Then Appendix A's claim is that the first sector of each quadrant is the mirror of the third sector of the next. Check it against the table:

- Q1 first is earth-from-fire; Q2 third is fire-from-earth. Mirror.
- Q2 first is air-from-earth; Q3 third is earth-from-air. Mirror.
- Q3 first is water-from-air; Q4 third is air-from-water. Mirror.
- Q4 first is fire-from-water; Q1 third is water-from-fire. Mirror.

Four for four, and the fourth closes the circuit from Water back to Fire — which answers a question I had raised and left hanging, whether there is any actual connection running from Q1 back to Q4 rather than only forward. There is. The loop of reflection runs the opposite way to the loop of rotation and closes at the same joint.

That accounts for eight of the twelve sectors. The remaining four are the seconds, and they pair too, but with the *opposite* quadrant rather than the next:

- Q1 second is air-from-fire; Q3 second is fire-from-air. Mirror.
- Q2 second is water-from-earth; Q4 second is earth-from-water. Mirror.

So all twelve sectors fall into six mirror pairs, none left over and none doubled. Firsts pair with the successor's thirds, adjacent around the wheel; seconds pair with the opposite quadrant's seconds, across the diagonal. Written out with the identifications this document has made:

sector	mirror partner	pairing
number line (<i>earth-from-fire</i>)	classical repositioning (<i>fire-from-earth</i>)	adjacent, Q1–Q2
2D area (<i>air-from-fire</i>)	interference (<i>fire-from-air</i>)	diagonal, Q1–Q3
3D volume (<i>water-from-fire</i>)	capacity (<i>fire-from-water</i>)	adjacent, Q4–Q1
separation (<i>air-from-earth</i>)	entanglement (<i>earth-from-air</i>)	adjacent, Q2–Q3

sector	mirror partner	pairing
constrained volume (<i>water-from-earth</i>)	light cone (<i>earth-from-water</i>)	diagonal, Q2–Q4
amplitude (<i>water-from-air</i>)	admissible futures (<i>air-from-water</i>)	adjacent, Q3–Q4

What this is worth, stated carefully. It is not new information. The pairing is forced by the ordering rule; anyone with the rule and a pencil gets the same six. Strictly it is bookkeeping, and I would rather say so than dress it up.

What is not bookkeeping is that all six pairs *mean* something, and nothing made them mean anything. The first difference ever produced is mirrored by the law under which all differences behave alike. Classical separation — how far apart two things are in real space — is mirrored by entanglement, which is precisely the failure of separation. A single allowance is mirrored by the volume of everything the whole could admit. Bare Euclidean room is mirrored by room considered as permission. And the first area ever made is mirrored by the very area the Born rule squares.

Not one of those correspondences was designed. They fall out of a rule that was fixed two versions ago to settle a quite different question, and the identifications they connect were made one quadrant at a time without any of this in view. That makes the mirror structure the strongest corroboration the ordering has — and the only one that owes nothing to the zodiac, since it derives from the rule rather than from the chart the rule was checked against.

Three consequences worth recording.

It is a constraint on any loop, not just this one. The pairing follows from the ordering alone, so every fractal iteration above the base loop has its own six pairs in the same arrangement. That is something a higher tier can be held to.

It gives the Born rule argument a second corroboration it did not have. Part V already notes that Air’s area is the fire-facing rung and that fire is the modality of fact. The mirror adds that the same area is the partner of the first area the loop ever produced — the 2D area of Fire, made by reflecting the number line. Interference is not an exotic quantum object that happens to be quadratic. It is the Air-quadrant instance of the operation that made area in the first place, and the mirror says so structurally.

It sharpens what the Water identifications are claiming. Capacity pairs with volume, and the pair reads as one thing seen twice: how much room there is, as Euclidean fact, and how much room there is, as permission. If the light-cone identification is the one to be most careful with — and it is — then it is worth noting that its partner is the constraining of volume at 5De, the Machian rung, which is the quadrant’s first genuine fact and the least speculative thing in Earth. A referee attacking c-as-shape has to explain why it mirrors the Machian rung so neatly.

What the powers of *i* actually do

The sign is not decoration. In plain words, what each rotation does to a thing:

- **i** — the outward becomes inward. A thing acquires an interior it did not have. Its content will not sit flat in the space its relations allow, so it turns into the orthogonal and holds the surplus inside. This is mass, and it is what boost means.
- **-1** — the inward becomes the whole of it. The negation does not annihilate; it reverses which side the thing is on. What is negated is the thing's outward address. It is still an existent and still doing something, but there is no longer a *where* at which it does it.
- **-i** — the inward is given back. The rotation undoes the first exactly. The interior is surrendered and what remains is outward only: a void with no inner value determining it, though still constrained by the real objects around it. A hole with a shape.
- **+1** — what remains comes back outward, but not as what went in. Two negations restore the sign; they do not restore the thing. What comes out is not an object but a fact, and the coming-out is itself the next dimension.

Everything in Part IV unpacks those four sentences.

The Mandala as guide

Because I took an interest in astrology — itself a long tale, full of characters, some of whom I know better than others, all of whom had interesting lives — I was able to construct the Mandala, the Alethiogram, which I take to be a map of Heaven: of everything that can be, and its relations, in the abstract. It has been the guide to discovering this story. A Golden Compass, as Philip Pullman might say. It does not replace science; it points the way. From it we can see the big picture in the abstract before working out the detail, and we should always keep in mind that the abstract is not the detail.

What the Mandala shows is that the rotation into boosted reality — the physical world, gravity and the rest — is not the last rotation. There is a complete circle and we have only seen half of it. The other half is a reflection of everything foregoing, and a rotation, and an involution. Its consequences as abstraction are easy enough to extrapolate from the first half of the map, but they are not at all obvious until you look closely at what is already in front of you.

The first thing to see is that the next step is another turning inward, another intrinsic rotation like the one that produced boost. But by then we already have 6D: three of Euclidean space overlaid by three of boosted space. Rotation takes us from boosted space to inner space, which is a rotation from reality but a subtraction from factuality. That is easier to grasp mathematically than as the establishment of Quadrant 3, the air quadrant. What we can see is that there is a plane of reflection between Q1 and Q4 on the left side and Q2 and Q3 on the right, just as there is a plane of reflection between Q1 and Q2 on the top half and Q3 and Q4 on the lower. And that there is a rotation running addition, boost, subtraction, un-boost, addition, bringing the whole of Existence full circle.

Understanding the transforms this way makes it clear that the transforms are always the same wherever we start from, and that the consequences differ across each boundary.

The transform from Q2 to Q3, earth to air, strips away the externality of difference — the objectivity that comes from extrinsic relationships of wholeness — leaving the intrinsic reality. That intrinsic reality no longer recognises the extrinsic hyperdox of boost. It is the full

intrinsic state of all the parts and the whole they form, as an internal collection of parts and relations, devoid of the Q1 relations that enforce the facts of Q1. Q2 has to abide by those rules. Q3 looks like it must be a law unto itself.

What is Q3 if it is devoid of Q1? Is it entirely devoid of Q1? What has been removed by turning inward from reality?

Factuality and reality do not present a dualism of being. They are all the same stuff, emergent facts. But the perspective of each quadrant gives a very different view of what is. In ideal factuality, triangles emerge from the folding of numbers, and that enables rotation as an afterthought to reflection: reflection gives a dimension at this stage, and the second dimension allows rotation about a point. The triangles are complex numbers; there is no difference except in how we now describe them; and thanks to the property credited to Pythagoras we get circles from rotation as well as squares on the hypotenuse.

The fourth dimension, boost, is only possible with those triangles. So it is appropriate to describe boosted space in that manner. But the triangles so formed, and every other triangle, share the same fact of triangularity — and here I mean the subset of triangles with an orthogonal corner, thanks to folding. The fact is replicated at every turn and remains the same fact, which justifies Leibniz's Law as much as the rational nature of number does. The nested medial triangles of Part I are the same point made with a pencil: every one of them is similar to every other, and all of them share one centre.

Objective facts form truths, and truths have a specific nature. They are virtual. Part of that nature is to be unchanging once formed. Another part is to behave in precisely the same manner in every instance, which is really just the claim that anything will behave according to its nature: if you want different behaviour you must change the nature of the individual thing.

Internality, then, is the nature of Quadrant 3, and it is the internal perspective that comes with the rotation and reflection from Q1 and Q2. The process is one of turning inside out. A real triangle composed of three real objects has the fact of being a triangle; turning it inside out gives an inner triangular perspective with no external context. The external properties — virtual relationships including relative size and the rest — are gone, subtracted by the involution. The internal perspective is of an island, which in this case is three points. Individual, and largely just defining a void.

That void, the defined inner space, is Quadrant 4 — Individuality — and the rotation that comes with reflecting Q1 and Q3 subtracts Q2. Internal space leaves very little except a perspective on emptiness — until something is done to populate it and give it form, and that is Tier 2's business. For now: Q4 is reached by subtracting reality, and the process mimics decay, the weak force, which properly is seen as emergent in Q2. Q4 is properly just the void, but it is defined by its now-missing shape, and it has the potential that if it can be reshaped by the internality of Q3 then it may, as some kind of fact, be added to Q1 and discover the eternal life of factuality. That last clause is the door the tier above comes through, and Part V says what happens when it does.

Q3 and Q4 each emerge in their own way, and that way is as a reflection of Q1 and Q2. Whatever in Q1 can be boosted into Q2 can also be negated from Q3, defining a shaped void that is the possibility of Q4 reflecting the facts of Q1, and allowing the manifestation of fact as internal perspective.

By this stage of the story the same four states are still at work: the orthodoxy of Q1 and its collection of all facts, the hyperdoxy of boosted reality in Q2, the hypodoxy of internality in Q3, and the anodoxy of individuality in Q4.

Part IV — The fourteen steps

The Fire quadrant

Fire is sameness; it is the eternal past; it is the orthodox. Numbers are numbers, Euclidean objects are Euclidean objects, facts are facts, and truth is the identity of facts. It is not that the region cannot tolerate difference — difference here is an absolute truth rather than a relative one, because there is no real space for it to be relative in. That makes the region one of objective wholeness, from which all derived objective parts must be constrained and governed.

1. $\circ D \rightarrow \circ D$ — the involution

It is probably a category error to treat nothingness as a possibility. But we would still want to know why nothingness is impossible, and the reason can only be got at by looking at what nothingness would have to be like. It would necessarily be complete, entire, unified, and have a root that was 1.

Nothingness turned inside out by the impossibility of being nothing and yet being whole, complete, entire. That makes being a point. A point cannot be rotated — or rather, rotating makes no difference to it; and if it were involuted it would go back to nothing. So its appearance is an imaginary paradoxical rotation, an involution: a nothing is paradoxically something, and it involutes by a rotation that adds. The being is then inside of nothing and outside of nothing, because involuting either way reverses the being and the non-being. A paradox, but an ontological one — not about belief and meaning but about *dasein*.

Being is a point, and because it is a turning inward and outward from nothing, it is **idiotropic**: a kind of navel-gazing. The large issue is that having sprung into existence there is still no dimension, only the possibility of a direction away from the point.

There is a root to nothing, its Tier 0, an intrinsic inner nature, and that also is unity. There is also a root to Unity, another potential dimension that emerges only when the extrinsic dimension does.

This is the seed step, and the one place where a quadrant does not receive its line from a turn. There is nothing prior to carry across; the involution can deliver a point and no more. Every later quadrant is opened by a turn that arrives already carrying content, and receives its line with the turn. Fire is the exception, for a reason rather than by oversight.

It is also the template for every boundary the architecture will ever cross, including the one at the top. A thing that cannot be what it is and yet be whole turns inside out, and the turning is the emergence. Step 1 does it to nothingness. The closure at step 14 does it to factuality. See *What the closure hands upward* in Part V.

Ruling on the involution's bookkeeping. v003 left open whether the involution counts among the four powers of *i*. It does not, and the reason is in the paragraph above. A rotation

needs a plane to turn within. At this step there is none — that is precisely why the involution delivers a point rather than a line. It is an involution in the strict sense, its own inverse, and not a rotation at all.

What the original was tracking in saying “the i squared becomes a -1 ” is nonetheless real. The Fire-to-Earth turn carries the seed involution forward inside it, so its *content* is the double involution that first creates something and then turns it inside out. Composite in content, i^1 in power. Both statements stand once the distinction between the two is drawn, and only on this reading do four rotations close the loop rather than five.

2. 0D → 1D — earth from fire: the number line

The hyperdoux of being causes the emergence of two-ness as the nodes of nothing and something, a partition of the unity of being, commencing the number line. This is the beginning of difference — the first reflection of one point with another — and this reflection causes, is caused by, or *is* the relationship between something and nothing. Every single thing populates that line, either partitioning it further as continued repeating reflection of unity against the most recent partition, or as imaginary turns inward, or as reflections outward of the whole line.

Here we get an *ausrichtung*, an outward heterotropic dimension: a line that properly speaking is internal to the unity that is being, but before anything emerges on that dimension there is just unity.

As the line emerges, populating unity with partitions, it populates the roots of unity, because each partition is both a whole like unity, an individual, and a unique partition of unity. They are all numbers, and so overlay one another, and that is what enables mathematics.

The reflection is toward earth and its driver is hyperdoux, which is why the line never finishes: the over-determination of being-and-nothing is renewed against every new partition. Each new integer is both a resolution of paradox and the creation of a new one. The final integer partition is always one more than the quantity of integers in Existence, which puts the cutting edge at the opposite end of the spectrum from Cantor’s diagonal argument — and which is the same claim made in Part I as the tortoise laying his own track.

3. 1D → 2D — air from fire: area

Just as the point defined a possible line away from it, the line defines a possible second direction, and reflecting the whole line along that second extrinsic dimension gives area. (This is the latent-orthogonal rule of Part I doing its first piece of work.) As the number area emerges alongside the number line, the roots of number extend as square roots.

The reflection is toward air and its driver is hypodoux: the numbers can now fold through possibility, which is the natural consequence of the separation that hyperdoux produced. This is the step I have elsewhere labelled air from fire in terms.

There is a worked example here, and it is the cleanest one the theory has. Four is necessarily both 1×4 and 2×2 . These are not two descriptions of one number; they are two structurally incompatible organisations of unity. 1×4 lives in one dimension. 2×2 requires two. Holding both true at once requires the second dimension to exist for them to live in. Three has no such problem, being only 1×3 . Four is the first paradox-square, and it is where the forcing can be watched happening without any prior commitment to physics — which is exactly what

makes it the example to lead with in print. Its companion, and the one to lead with second, is the tetrahedron of Part I, where adding a third direction to a self-similar recursion produces an octahedron that nothing in the two-dimensional case predicts.

This area is also the mirror partner of interference at step 9, which is worth knowing before the Born rule argument arrives. See *The loop of reflection* in Part III.

4. 2D → 3D — water from fire: volume

In the same fashion, reflecting area along the third extrinsic dimension produces volume, and the roots of number extend again — as cube roots, not square roots. (voo2 and the original both say “square roots” twice; the second is a slip.)

Volume, an empty space, is conveniently labelled water, as in the Platonic elements. The reflection is toward water and its driver is anodox: hypodox reflects again because it can, and in doing so defines the void that lets it be and change.

At this stage we have a Euclidean realm of numbers and mathematical objects, with the only generative contradiction being the ever-increasing quantity of partitions, caused by the hyperdoh of unity reflecting against the most recently generated partition.

An inquisitive mind might ask whether the ticking of new numbers is Time. It is not, and Part V says why.

The Earth quadrant

Earth is difference; it is the present; it is the hyperdoh. Absolutes with relative differences: masses are mathematical objects that possess boosted differences. What emerges here is the reality of space — three-dimensional space with boost, which is Einstein’s mass-energy equivalence with no involvement of time yet. That makes the region one of objective difference: the real parts that can form real wholes.

5. 3D → 4De / 1Di — the first turn, fire to earth

Reality breaks out of the Euclidean realm once the numerical objects run into a hyperdoh. Some numbers are both 3D and 4D — they fold both ways — and one way the differences between them as 4D are less than the differences between them as 3D. The resolution is a necessary imaginary dimension, the boost of a rotation into the orthogonal.

At the same time as creating reality as boosted space within Euclidean space — an intrinsic relationship — an extrinsic fourth dimension emerges as external to the imaginary reality of boost. This 4De, as against 4Di, is the first dimension of Reality, which ironically we should probably call 1Di, on the grounds that the dimensions before it were perhaps not quite so imaginary, or at least not solely so.

What the turn erects (1Di): mass. Fire’s denied fourth reflection was reaching for earth, which is difference. Turned inward, difference becomes the boost a thing carries in itself. The force is the strong force together with gravity.

What the turn opens (4De): the line of Earth — air from earth. Separation in real space: the bare fact of being apart, and how far apart. Distance and relative position. Its driver is hypodox, and it is under-determined in exactly the sense the word requires — it

settles that things are separated and by how much, and settles nothing about what they will do about it.

With each increase in dimensionality the available transforms increase, and they begin to perform very different behaviours as the count rises. Ultimately we reach the situation in the large language models, where concepts of many dimensions such as royalty and gender can be added and subtracted: King – Man + Woman = Queen. Which makes me wonder whether King × Queen = Heirs, and Heirs + Male = Prince.

6. 4De → 5De — water from earth: area

The second reflection of the quadrant, toward water, and its driver is anodox — the void.

Two separations reflected against one another give an area, and the area here is the constraining of volume: how much room the whole allows a massive thing to occupy, given everything else. It is a relation between a thing's separations and the void those separations define. This is the first genuine fact of the quadrant, because it is the first rung that is a relationship rather than a magnitude, and it is where the Machian character of the account enters. The room a thing has is a fact about everything else, not about the thing.

Its mirror partner across the diagonal is the light cone at step 12, which is the same question — how much room is there — asked of permission rather than of matter.

7. 5De → 6De — fire from earth: volume

The third reflection, toward fire, and its driver is orthodox — sameness, and therefore law.

Reflecting the constrained area along a third direction gives the volume of joint repositioning: every massive thing being repositioned by every other, all alike, all under the same relation. This is classical mechanics considered as an object rather than as a description, and the *alike* is the point — the reflection is toward fire, the modality in which all instances of an indiscernible feature are the same instance.

It is also the end of the quadrant's rope. A fourth reflection would return to air, the separation the first reflection already took, and the duplication cannot be held flat: everything must move relative to everything, and in real space the demands cannot all be met at once. Something must give, and what gives is the outward route.

The Air quadrant

Air is separation and inner change; it is the imminent future; it is the hypodox. Its objects are the inner consequences of the previous quadrant, and it is the quadrant of what electromagnetic change does beyond the material. Transfers of value across space must convey information: the thing transferred has its own data, and multiple such transfers convey the information inherent in the relations of their sources. Air is therefore the inner space of space, and because combination happens at the destination it is internal, though composed of parts like the reality it reflects. That makes the region one of internal difference, and it is the quadrant named Internality.

The objects of this quadrant cannot be seen, and the reason is different from the one that will apply in Water. Air-quadrant objects are unseeable because there is **nowhere to look**. Seeing requires an outward address to point at, and the negation has removed it. They are

not hidden behind anything and they are not too small; there is no place at which the looking could be done. All that can be had of them is what they do when they arrive back on the real axis, which is why every measurement is a report of a landing rather than of a flight.

This is internality in the sense set out in Part I — a fact about which way a relation faces — and it carries no implication about experience. The objects below are amplitude, interference and entanglement. They are physics.

8. $6De \rightarrow 7De / 2Di$ — the second turn, earth to air

The second rotation, $i^2 = -1$, and the minus does real work. The sign-flip strips squareness — area — off one object and deposits it on another. Because these objects have no classical position there is no literal here and there, only a direct relation of change between them. Philip the photon is this step, and what he carries is momentum. The force is electromagnetism.

What the turn erects ($2Di$): momentum. Earth's denied fourth reflection was reaching for air, which is separation. Turned inward, separation becomes a commitment to change relation, held internally with nowhere to point at.

In Earth a thing's difference from its neighbours was its position, an outward fact available to anyone. Here the outward address has been turned inward, and what is left is the same fact on the other side.

This is why position and momentum cannot both be sharp, and it is not the strange result it has always been presented as. Position is an extrinsic dimension of Earth. Momentum is the intrinsic dimension of Air. They are not two properties of one object competing for precision; they are one relation read from opposite sides of a ninety-degree turn, and a thing cannot be fully on both sides of a turn at once. Heisenberg was reporting the geometry of the break.

Note also the shape of this transfer, because it is the model for every crossing of a two-dimensional divide anywhere in the architecture. An area is taken off one object and put on another, and nothing travels, because there is no here and no there for anything to travel between. Wherever something later appears to act across a gap it cannot cross, this is the move to look for first.

What the turn opens ($7De$): the line of Air — water from air. Air reflected toward water is separation reaching toward the defined void: a part asking what it may be, of the whole that contains it. Since these objects have no position, the only extension available to them is not spatial. What extends is the extent of the claim — how much of the whole a given arrangement of parts may take up. This is a genuine one-dimensional magnitude, more or less of it, and it is entirely objective. It is not our estimate of a chance and it is not a bookkeeping device; the whole really does allow this arrangement so far and no further, on the strength of the facts that constitute it.

This is what is elsewhere called **amplitude**, and the identification is close enough that I will use the word. But the reading differs from the usual one. Amplitude is normally introduced as an unexplained quantity whose square happens to give a probability, and the squaring is a postulate nobody likes. Here amplitude is the line of Air, the bare extent of an allowance, and its driver is anodox — permission, the modality of the void. The squaring turns out to be a dimensional fact rather than a rule imposed from outside.

9. 7De → 8De — fire from air: area

The second reflection, toward fire, and its driver is orthodox — fact.

Just as the point defined a possible line away from it, and the line an area, so an allowance defines a possible second allowance to be reflected against. Air reflected toward fire is separation reaching toward sameness: one allowance asking of another whether they are, in the relevant respect, the same. Two lines reflected produce an area, and the area is where the two agree or disagree. Where they agree the area is large; where they disagree it cancels.

This is **phase**, and the area is **interference**. I take the identification to be exact, and it pays a debt outstanding since the twenties. The argument is in Part V; the structural point to hold here is that this reflection, alone among Air's three, is directed at fire — and fire is the modality of fact. The area of Air is the fire-facing rung of the quadrant. That is why what can be observed of an Air-quadrant object is an area and never a line.

And it is the mirror of step 3, the first area the loop ever made. The diagonal pair runs Q1's second sector against Q3's second sector, air-from-fire against fire-from-air, and the identifications on the two ends are 2D area and interference. Interference is not a quantum peculiarity that happens to be quadratic. It is the Air-quadrant instance of the operation that produced area in the first place, and the mirror structure says so without being asked.

The break into this quadrant was *i* squared, and the note on it has always been that the sign-flip strips squareness off one object and deposits it on another. Squareness — area — is the characteristic object of this sector. The break and what the break opens onto are one structural fact seen twice.

10. 8De → 9De — earth from air: volume

The third reflection, toward earth, and its driver is hyperdox — over-determination. This is where the quadrant breaks, and unusually the driver and the break coincide.

Air reflected toward earth is separation reaching toward difference: the possibilities of two or more parts taken jointly rather than severally. Reflecting the whole area along a third direction produces a volume, and the volume here is a joint allowance — what the parts may do together, which is not the sum of what each may do alone.

This is **entanglement**, and the reason it is entanglement rather than a mere combination is that the joint volume is over-determined. The whole determines the parts to more than the parts can severally carry. Each part alone has its own allowance; together they have a further one, and the two cannot both be satisfied in the dimensions available. That is a paradox in the strict sense — determinate on both sides, actual rather than descriptive, and unpreventable, since no rearrangement within the quadrant will flatten it. Bell was measuring the over-determination.

Its mirror partner is separation at step 5, the line of Earth. That pairing is worth a moment: the sector that says how far apart two things are is mirrored by the sector in which being apart has stopped meaning anything. The mirror was not arranged to say that. It is what the ordering rule produces.

So Air's third reflection is doubly the break's cause: it exhausts the targets like every third reflection, and it is itself hyperdox-driven. What happens next is forced.

The Water quadrant

Water is internal wholeness — Individuality; it is the distant future; it is the anodox. It is formed by the combinations of changing information in Air, and in one sense it reflects Air — they are two sides of a coin, but the flip from one side to the other is a rotation. Removing the parts leaves just the fact of the whole to populate the quadrant, and that defines both what something is and what it is not.

Its objects are the least touchable of anything in the scheme, and the reason for their invisibility is not the reason that applied in Air. Air-quadrant objects could not be seen because there was nowhere to look. Water-quadrant objects cannot be seen because **there is nothing there**. They are room, not occupant. What they contribute is not content but permission, and permission has no surface.

There is a second reason, geometric rather than categorial. By this point in the loop an object carries so many dimensions that its extension in any three of them has become vanishingly small. The higher-dimensional content hides inside the lower-dimensional container, and the container is nearly all that is left to look at. The original puts the onset precisely: extrinsics beyond 5De begin to shrink in absolute terms, not merely in the relative comparison of boost, and area tails away at 7De. So the thinning begins at the line of Air and is all but complete by the time Water opens. If you want a picture, take one of those many-pointed stars whose points grow narrower with every additional direction, until the thing is almost all centre and almost no reach. The distant future is not concealed from us. It is geometrically almost absent.

11. 9De → 10De / 3Di — the third turn, air to water

The third rotation, $i^3 = -i$, undoes exactly what the first one did. Where i took a thing's surplus content and turned it inward as boost, $-i$ takes the inward value back out and does not replace it. This is the **un-boost**, and it is the only genuinely subtractive direction in the architecture. Everything else adds, or moves, or turns; this alone removes. The force is the weak force.

It should sound strange, and it does. We are so habituated to treating reality as irreducibly real that a step which simply takes reality away looks like cheating. But it follows from the as-if commitment, on which the correct mathematics is entitled to perform operations that look like magic, because there was never any stuff there to conserve. What is conserved is the fact, and the fact of the removal is as good a fact as the fact of the arrival.

What the turn erects (3Di): relinquishment. Air's denied fourth reflection was reaching for water, the defined void. Turned inward, the void becomes the axis along which a thing can cease to be the thing it was. Not move, not change state — cease. This is loss of mass, and the weak force fits it exactly: a particle's identity is not rearranged but surrendered.

What the turn opens (10De): the line of Water — fire from water. What is left behind when the inner value goes is not nothing. It is a void no longer determined by any inner value of its own, but still shaped by the real objects around it: a hole with a shape. This is the object Part I met at the outset as the centre of a triangle — determined entirely by what borders it, occupied by nothing, and invariant under the replacement of everything that determines it. Water reflected toward fire is wholeness reaching toward sameness — the void

asking what is true of it — and its driver is orthodox. The void's one truth is that it is not nothing; it has extent without content. So the line of this quadrant is **bare capacity**: the sheer amount of room there is for something to be, considered apart from anything being in it.

Capacity settles nothing about what will occupy it; it settles only how much occupying there could be. And it is objective — the room a region affords is a fact about the constraints of the real objects surrounding it, not about anyone's ignorance of what will happen there. If a physical name is wanted, this is the vacuum, taken as a quantity of permission rather than as a sea of anything.

Its mirror is step 4, the volume of Fire, and the pair closes the loop of reflection from Water back to Fire. Euclidean room and room-as-permission are the same question asked at the two ends of the circuit.

12. 10De → 11De — earth from water: area

The second reflection, toward earth, and its driver is hyperdoh.

Water reflected toward earth is wholeness reaching toward difference: the comparison of what is possible here with what is possible there. Two capacities reflected give an area, and the area is a boundary — the surface separating what can be reached from what cannot.

This is the shape of the possible, and I take it to be what the **light cone** is. That is a considerable claim and it should be put precisely. The usual reading treats the cone as a consequence of a speed: light goes so fast, therefore this region is reachable and that one is not. On this account the priority runs the other way. The cone is the area-object of the Water quadrant, a surface in the space of permissions, and c is its shape rather than anything's velocity. Nothing is travelling at the boundary; the boundary is where the whole's permission runs out. Light happens to sit on it because light has surrendered all its inner value and so has nothing left to hold it inside the surface. The massless case is the boundary case, arrived at from the direction of the un-boost rather than by taking a limit.

That the driver here is hyperdoh is not incidental. A boundary is an over-determination made geometric: it is where two capacities cannot both be satisfied, and the surface is the record of the disagreement.

And this is the most thoroughly unseeable object in the scheme. It is an area with nothing on it. There is no material at the boundary, no substance to reflect anything back at us, and yet it decides what may happen everywhere. It can only be deduced, which is what this exercise was for.

Its mirror partner across the diagonal is the constraining of volume at step 6 — the Machian rung, and the least speculative fact in Earth. Anyone minded to attack c -as-shape has to say why the two sit in the same mirror pair.

13. 11De → 12De — air from water: volume

The third reflection, toward air, and its driver is hypodoh. The last extrinsic dimension in the base loop, and the one that forces the closure.

Water reflected toward air is wholeness reaching toward separation and change: the whole considered as all the changes it could undergo. Reflecting the boundary-area along a third

direction gives the volume of the possible — every admissible future of the whole, taken together. This is King William the Wave at his full extent: the whole-without-parts, holding every arrangement its parts could be constrained into.

Its mirror is amplitude at step 8: a single allowance against the totality of allowances, the one and the all, at the two ends of an adjacent pair.

The quadrant is now out of targets, and the fourth reflection would return to fire. That duplication is over-determined for a reason easy to state and impossible to evade. Every admissible future claims the same room. They are not laid out side by side in some larger space where each may have its own province; there is only the one whole, and each possibility is a claim on the whole of it. Taken singly, each is fine. Taken jointly — and they are joint, because they are possibilities of one thing — they determine the whole to more than one settled state at once, which no whole can be.

That is the terminal paradox of the base loop, and no rearrangement within the four quadrants will resolve it. Every direction has been used: volume in Fire, boost in Earth, area in Air, capacity and boundary here. There is nothing left to turn into except the real axis.

The closure

14. 12De → 4Di — the fourth turn, water to fire

The last rotation brings us to $i^4 = +1$, and two things happen at once, which is what makes this step different in kind from the three before it.

The first is that **the phase is settled**. Four right angles owe no net rotation, so the closing step costs nothing. It is force-free, and it needs no carrier — there is no energy to transfer because there is no particle, only the as-if appearance of one. This is the meta-level analogue of the photon: a quantum of transfer that adds here and subtracts there with no residue. Its base-loop instances are the ones that have always looked impossible on a carrier account — collapse, spin, the immediacy of entangled correlation. They look impossible because a carrier is being sought for a step that never needed one.

The second is that **the return is not to where we started**. The +1 is a displacement, and what it erects is the seventh dimension — seventh in the count of directions genuinely erected, which is the three Euclidean ones and the four turns, as set out under the governing rule in Part III. Water's denied fourth reflection was reaching for fire, which is fact; and the content of the step is the inversion of Air. Air was the inner layer of an object, its probabilistic possibility held privately. The water-to-fire turn takes that inner fact and makes it an objective outer one. The whole-without-parts collapses into a single logged fact. The internal becomes objective. **A thing becomes true.**

The intrinsic dimension erected here, 4Di, is therefore becoming-true itself: actualisation considered as a direction rather than as an event. This is the only one of the four turns whose product is a fact rather than a property. The first gave mass, the second momentum, the third loss; the fourth gives a truth, which is a different sort of thing altogether, and is why it can leave the loop and take its place in the eternal past.

What the closure deposits, which is two things and not one

Revised in v007. Up to v006 this step was read as depositing one thing, the law. It deposits two, and they are the two halves of Leibniz's Law arriving together.

What is alike across instances becomes law. In the base loop what becomes true is the physical universe becoming true as the actual laws of nature. A law is a sameness — all similar objects behave alike under like circumstances, which follows from all electrons being one electron in the respect in which they are indiscernible. Law is not a soft thing we layer on top of the world. It is the seventh-dimensional fact this loop deposits, fully determined, and it is in the past, which is where all facts are.

What is unlike is still two, and that is deposited as well. Black's iron balls are not one ball, because they are in different places and spatial position is always discerning. What closed here was a *particular* whole, with a particular interior, and the fact of its having been particular comes down with the law. Call it the fact of individuality: *the fact of internal perspective, but without the internal perspective per se*. It is a fact about an inside, carrying no inside — the silhouette left when Water's possibility is inverted into Fire's factuality, read for what it is a negative of.

Since v008 that name is no longer a coinage but a reading-off. Water is Individuality, the quadrant of a void defined in terms of a self, and what the closure sends up is the fact of that. The quadrant that hands the surplus upward is the quadrant the surplus is named after, and the two had been arrived at separately.

Both halves are the same deposit read on its two sides, and the second half had gone unnoticed because the first is the one physics cares about. It matters because Fire can accommodate the first and cannot accommodate the second, and that is where the next tier comes from. The argument is in Part V.

Being a whole, an object of Water is factual, and it is simply orthodox for that fact to add to the totality of facts by making the final rotation. Getting into Water from Air is the hardest thing to imagine, and as a change it resembles physical decay. The step from Water to Fire is a straightforward addition of fact, albeit an inversion, like a photographic negative — the water element being the space of what is not, as defined by the silhouette of what is.

Part V — What comes out of it

The Born rule

This is the one place in the base loop where the scheme produces a result that mainstream physics has to postulate, so it deserves stating properly rather than loosely.

For anyone meeting it cold: Born's rule, from 1926, says that the probability of finding a quantum system in a given state is the square of the modulus of its amplitude — the amplitude multiplied by its own mirror image, $\psi^*\psi$, written $|\psi|^2$. Nobody has ever derived it from the rest of the theory. It is bolted on, and the bolt has been visible for a century.

The argument in its cleanest form

Within a quadrant the three reflections go line, area, volume — the same three-fold that furnished the Euclidean realm from the point. In Air the line is the allowance: the bare extent of a possibility's claim on the whole. An allowance is the potential for a new dimension, and potential is an incomplete state — just one end, so there is no relating going on.

That is the argument in one move. An allowance taken by itself is a single node. It is not yet a relationship, and on the foundations a fact is a relationship: a single node with nothing to stand against is not a fact but a term. So the first thing in the Air quadrant that qualifies as a fact at all is the area — the allowance reflected. And if only facts can be observed, then what is observed is never the allowance but always the area. The observable goes as the square, not because a rule says so, but because the line was never a candidate. You cannot photograph a line; it has no surface to catch the light.

The load is carried by “facts are relationships”, which is foundations_v003.md §3 and did not have to be invented for the purpose. (v002 cited §2; the correct section is §3.) The second load-bearer is the ordering of Part III, which is what puts the allowance at the line and the interference at the area rather than the other way about; the consequence is spelled out there.

Author's ruling, carried from v002: “Yes, this is entirely correct.”

The corroboration the ordering supplies, now twofold

v002 rested the argument on a second prop: that squaring is what happens at step two of a three-fold, because step two is the non-determined rung of a fixed ladder. That prop is gone, because the ladder does not run that way — the modality of a reflection is the modality of its target, and it rotates between quadrants.

What replaces it is better, and as of v007 there are two of them.

First, the fire-facing rung. Air's second reflection is directed at **fire**, and fire is the modality of fact. The area of the Air quadrant is not merely the first rung that happens to be a relationship; it is the rung whose target is factuality. So the claim that what is observed of an Air-quadrant object is its area is underwritten twice over: once by the general principle that facts are relationships, and once by the specific structure of this quadrant, which reaches toward fire exactly there and nowhere else.

Second, the mirror. The loop of reflection pairs Air's area with Fire's area — fire-from-air against air-from-fire — across the diagonal. Interference is the mirror partner of the first area the architecture ever produced, the one made at step 3 by reflecting the number line. That says something the fire-facing argument does not: interference is not a quantum object that happens to be quadratic, it is *the same operation* as the one that made area in the first place, running in a quadrant where the things being reflected are allowances rather than integers. Squaring is what area-making is. It was never a rule.

Neither corroboration existed before the ordering was settled, and the second did not exist before v007. I rate the pair the most useful thing to come out of the last three passes.

The correction that improves the argument

vo02 said the area comes from reflecting one allowance against another. That gives interference, but it does not give the Born rule, because the Born rule is a *modulus* square — $\psi^*\psi$, not ψ^2 . The amplitude is complex, and squaring it would yield complex probabilities, which is nonsense.

The conjugate is a reflection in the real axis. So $|\psi|^2$ is the allowance reflected against *its own reflection*: a single line making an area with itself. That is exactly the operation the Maze uses at every step, which makes the modulus-square the purest instance of the reflection already in hand rather than an awkward extra. The two-allowance case then gives the cross-terms. One operation, both results.

Normalisation comes free on the same terms. The allowances are claims on one whole, and every partition is a partition of unity. That they sum to one is not a further postulate; it is what “of one thing” means.

One caution, recorded rather than hidden, and cross-referenced in both directions. In Part III I say that a node reflected against itself is not a relationship and so not a fact — that is the argument closing off the wrong reflection order, and it is also the argument making the line unobservable. Here I say that a line reflected against its own conjugate does make an area. The two are compatible only because the conjugate is a genuinely distinct node: a reflection in the real axis is a different position on the complex plane, not the same node named twice. The distinction is fine enough that a referee will press on it, and it should be made explicit in anything written for publication.

Why it is worth taking seriously

There is a fact about the quantum formalism that fits this uncomfortably well. Nothing observable in quantum mechanics is ever an amplitude. Every observable quantity is *bilinear* in amplitudes — quadratic, an area. Expand a superposition and you get $|\psi_1|^2 + |\psi_2|^2 + 2\text{Re}(\psi_1^*\psi_2)$: the Born probabilities and the interference cross-terms alike, and every term a product of two amplitudes. Not one term in that expression is a line. Physicists treat this as a curiosity of the machinery. On this account it is the shape of the quadrant.

Where it does not yet reach

Having a quantity that is positive, quadratic and sums to one does not by itself make it a *probability* in the statistical sense — a frequency one would see over many trials. That step is the genuinely hard one, and it is what Deutsch, Wallace and Zurek have been trying to close for twenty-five years without consensus. The account has a position on it — probability is the whole’s tolerance, and frequencies follow because repeated trials sample the same tolerance — but a position is not a derivation, and a referee will say so.

Gleason, and how to frame the claim in print

Gleason proved in 1957 that for Hilbert spaces of dimension three or more the Born rule is the only possible probability measure on subspaces. So the squaring is already forced — **given the inner-product structure**. That sounds deflating and it is not: it sharpens what the claim should be. Gleason derives Born from the inner product and *assumes* the inner product. What the reflection operation derives is the inner product itself, and why the space

has an area-forming operation at all. That is a deeper target and a more defensible one, and it is how the claim should be framed for publication.

One curio, recorded and not banked: Gleason needs dimension three or more, and each quadrant has three reflections. There is no argument connecting them, and it is better to say so than to dress it up.

Time

Fire raised the question of whether the ticking of new numbers is Time, and answered it with “I think not — there is no parallel process to compare it to.” That answer can now be stated as a principle rather than an intuition.

Time emerges only where there are parallel processes acting as clocks to one another, and that requires spatial difference. One process on its own has no time, because there is nothing to measure its change against. Time is not a highway the universe drives down. It is what shows up when parallel processes intersect and can be read off against each other.

Three things follow.

The numerogenesis production line has no time, because at the point where partitions are being churned out there is no second process and no space for one to run in. The ticking is a sequence, and sequence is not yet time. So the question is settled in the negative, and settled for a reason.

Time cannot emerge before Earth, because Earth is where spatial difference first exists. That places the emergence of time after the first turn, which is consistent with the note in the standing appendix that Q2 gives 3D space with boost “with no involvement of time yet” — boost comes with the turn, time comes with what the quadrant’s extrinsics then permit.

Time is not the past. The past is ordered in accordance with the chronological sequence of events, which is a different claim. On this point special relativity has been misinterpreted, its interpreters having been seduced by the apparent reality of space and time. And note that every quadrant contributes to that past, whatever its modality and whether or not time is anywhere in the neighbourhood: facts accumulate because facts are eternal, and there is one past, not one per quadrant. What differs between quadrants is the kind of entry, not the ledger.

Whether the production of numbers is linked to causation and change is a question I have not attempted to address, and I am content to leave it open. There are two candidate answers and I do not yet see how to choose between them. Either the churning of numbers is the prime cause, and physical causation is a macroscopic echo of the deeper compulsion where an anodox forces an orthodox resolution; or the churning is outside cause and effect altogether, numbers simply being the structural logic of Existence, with causation arising only once space is there to host it. Both are consistent with everything above. Flagged.

The double cover, and whether the loop closes once or twice

New in v004, and mine rather than the record. It is a question with a sketch, not a result.

Part II established two things about the transforms. Composing two reflections gives a rotation through twice the angle between their hyperplanes. And a spinor is an object that tracks which reflections composed the rotation rather than only the net motion of the vectors, with the consequence that a 360° vector rotation leaves the spinor at -1 and a 720° one returns it to $+1$.

Now put that against the architecture. Each turn carries a ninety-degree rotation together with the reflection of the whole prior space — my standing note on reflections says exactly this, that reflecting a 3D object reflects each of its constituent dimensions, so lines, areas and volumes all rotate with the strong emergence of boosted space. Rotation composed with reflection is improper: determinant -1 . Four of them compose to $(-1)^4 = +1$, so orientation is restored at the closure, which is what $i^4 = 1$ already told us. So far, so consistent.

The question the spinor material raises is whether that is the whole story. Determinant $+1$ says the composite is a rotation. It does not say *which* element of the double cover the composite is. The base loop composes twelve reflections and four turns, and whether the whole circuit lands at $+1$ or at -1 in $\text{Spin}(n)$ is a calculation, not a matter of taste. I have not done it. Doing it needs the twelve extrinsic reflections given explicit hyperplane normals and the four turns given their planes, which is more bookkeeping than I have attempted and is exactly the kind of thing that ought to be checkable.

Added in v007: the mirror structure of Part III may make that bookkeeping easier than it looked. The twelve reflections are not twelve independent choices of hyperplane; they pair into six mirror couples, and a mirror couple is exactly the configuration whose composite is a rotation with a known relation between the two normals. Whether the six couples compose more tractably than the twelve singletons is worth half an hour before attempting the general calculation.

Both answers are worth having.

If the circuit lands at -1 , the loop closes for the vectors but not for the spinor, and it must run twice to close in both. That would give spin- $1/2$ a structural origin: a spin- $1/2$ particle is what an object looks like when it sits in a structure whose closure is a double cover, and the famous 720° would be the second circuit of the Mandala rather than a peculiarity of electrons. It would also say why the expansion past the base loop has to go fractal rather than continue as further quadrant-breaks: the second circuit runs the same relational order over the richer substrate the first deposited, because that is what closing in the spinor requires.

If the circuit lands at $+1$, the base loop is self-contained, spin needs its account from somewhere else, and the fractal expansion needs its motivation from somewhere else too. Note that as of v007 the fractal expansion has a motivation that does not depend on this calculation at all — see the next section — so a $+1$ result is no longer as costly as it was.

There is a related loose end that this may tidy up, and I record the connection without claiming it. $i + i^2 + i^3 + i^4 = 0$, while $i^4 = 1$. Two true statements about the same four turns that seem to say opposite things. The reading worth testing is that the vanishing sum is the *net displacement* — nothing has moved, everything is back where it started — while the $+1$ is the *phase*. A spinor is precisely the object that distinguishes a system returned to its starting configuration from one that has been taken round a full turn to get there. If that is right, the vanishing sum and the unit fourth power are not in tension; they are the two halves of what the double cover keeps separate, and the conservation across a full turn is real.

I would not put any of this in print until the composition has been worked out. But it is the most promising thing the retrieved mathematics turned up, and it has the virtue of being decidable.

What the closure hands upward

New in v007. The argument in full is in tier_boundary_v002.md; what follows is the short form, because Tier 1 has to state it even though Tier 2 will develop it.

Up to v006 this document said that past the seven base dimensions the expansion goes fractal, wholes within a sector spawning their own loops over the richer substrate the loop below deposited. It said it and did not show it, and that was the largest hole in the account — a forced foundation with an unforced superstructure, which is precisely the complaint Part VI makes against everybody else. If the second tier merely *becomes available* once the first has run, then something in the architecture permits rather than compels, and a small creator has been smuggled in at the top.

It is closed as follows.

The closure deposits two things (step 14): law, and the fact of individuality. Fire can hold the first. Fire's dimensions are the three Euclidean ones, erected on a point, and what lives in them is number, shape and the identity of facts — things alike wherever instanced, with no interior and no perspective, because there is no real space for either. A law is a sameness and Fire is the modality of sameness. The deposit lands in the dimension made for it.

Fire cannot hold the second. The fact of individuality is a fact whose content is that there was an inside — an orientation, a from-here — and orientation is exactly what a Euclidean fact does not have. Fire can hold that such a fact *exists*, because Fire holds all facts and cannot decline one. Fire cannot hold *what the fact is about*, because it has no dimension in which from-here means anything.

That is a discrepancy, determinate on both sides, with no rearrangement available. Run it against the three conditions for a generative paradox:

- **Actual.** A fact of fact, not of description. It is not self-referential — it has external relata, namely the whole it was a whole of, and the twelve sectors that whole traversed.
- **Unpreventable.** The closure cannot be declined; $i^4 = 1$ is not a policy. And it cannot be absorbed by rearrangement, because every direction has been used: volume in Fire, boost in Earth, area in Air, capacity and boundary in Water.
- **Determinate on both sides.** The fact is definite. Fire's inability is definite — not vagueness about whether some Euclidean dimension might accommodate orientation, but the absence of any such dimension.

All three hold. A new dimension is forced.

A tier begins where a closure deposits a fact that factuality has no room for.

Three things follow and are worth having.

It is step 1 again. Nothingness could not be nothing and yet be complete; the impossibility turned it inside out and being was a point. Factuality cannot hold all facts and yet lack the

dimension a particular fact requires; the impossibility turns it inside out and a tier begins. Same mechanism, opposite ends of the loop, which is what a theory with one engine ought to look like.

It gives a criterion rather than a count. A closure whose deposit is law forces nothing, because law lands where law belongs — and that is the ordinary case, the fractal iteration, the loop running again over richer material. A closure whose deposit is individuality forces a tier. So tiers are numbered by regime, not by iteration, and the question of whether Tier 2 is the *immediate* next tier dissolves: there may be a great many loops in between, and they are loops of the first kind.

It says what the tier above is standing on. Its substrate is this tier's completed factuality *plus* the surplus that factuality could not hold. Not one or the other. That is why facts of the tier above have to root themselves in a Tier 1 garden — they have nowhere else to root, and they are up there because down here there was no room.

And it says what kind of thing the tier above is. This is new at v008 and it comes from the standing note *On the question of Reality*, which puts the distinction in one line: if Tier 1 is about what things are, about the ontology of Existence in its various forms, Tier 2 is about the behaviour of Tier 1 — not what is, but what it does.

Set that against the forcing argument and the two turn out to be the same observation approached from opposite ends. Fire is the ledger of what things *are*: number, shape, the identity of facts, each alike wherever instanced. What the closure sends up is a fact about what a particular whole was like from inside — which is a fact about a *doing*, not about a kind. Fire has no column for it. So the surplus factuality cannot hold and the subject matter of the tier above are one and the same thing, and the boundary between the tiers is the boundary between being and behaving.

The claim wants one qualification, and the standing note supplies it. Behaviour is not absent from Tier 1: partition, addition, subtraction, multiplication, division and rotation are all things facts do, and the note calls them natural mathematics. The distinction is not that Tier 1 is static but that Tier 1 takes *what there is* as its subject and treats behaviour as what facts can do, where Tier 2 takes behaviour as the subject itself. More dimensions confer different behaviour rather than different ontology — which is why Reality is not ontologically distinct from Factuality, and why the tier above is not a second kind of stuff either.

What that tier then does with the surplus is not this tier's business, and the temptation to say more is to be resisted. What Tier 1 is entitled to say is that the door exists, that it is forced rather than offered, what comes through it, and what sort of question waits on the other side.

What the loop has produced, and what it has not

End to end the base loop runs: nothing, whose completeness it cannot help having; unity; the number line; area; volume; then the first turn into mass, with separation, the constraining of volume, and joint repositioning; then the second turn into momentum, with allowance, interference, and joint possibility; then the third turn into relinquishment, with capacity, the boundary of the possible, and the volume of the admissible; then the fourth turn, which logs a fact and rises.

Fourteen steps. Twelve extrinsic dimensions, four intrinsic. Seven erected. Six mirror pairs. One law deposited, and one surplus that could not be.

	Fire	Earth	Air	Water
modality	orthodox	hyperdox	hypodox	anodox
determination	fully determined	over-determined	under-determined	non-determined
time	eternal past	present	imminent future	distant future
line	number line (<i>earth</i>)	separation, 4De (<i>air</i>)	amplitude, 7De (<i>water</i>)	capacity, 10De (<i>fire</i>)
area	2D (<i>air</i>)	constrained volume, 5De (<i>water</i>)	phase / interference, 8De (<i>fire</i>)	light cone, 11De (<i>earth</i>)
volume	3D (<i>water</i>)	classical repositioning, 6De (<i>fire</i>)	entanglement, 9De (<i>earth</i>)	admissible futures, 12De (<i>air</i>)
turn out	$i^1 \rightarrow 1\text{Di}$ mass	$i^2 \rightarrow 2\text{Di}$ momentum	$i^3 \rightarrow 3\text{Di}$ loss	$i^4 \rightarrow 4\text{Di}$ becoming true
force	strong + gravity	electromagnetism	weak	force-free

And the same twelve extrinsics read as six mirror pairs rather than as four quadrants:

sector	mirror partner	pairing
number line	classical repositioning	adjacent
2D area	interference	diagonal
3D volume	capacity	adjacent
separation	entanglement	adjacent
constrained volume	light cone	diagonal
amplitude	admissible futures	adjacent

What the loop has not produced is anything above physical law, and that is by design. Past the seven base dimensions the expansion does not continue as further quadrant-breaks. It goes fractal: the wholes that now exist within a sector spawn their own loops, running the same relational order over the richer substrate the loop below deposited. Consciousness, shared awareness, the content of a particular moral fact — these are the facts of inner change becoming raw material for the next turn, the self-becoming-true repeated at a higher substrate. The rails are laid by the base loop. The traffic on them is not Tier 1's business. What is Tier 1's business, and is now discharged, is saying why the traffic is compelled to run at all.

The touchstone was always going to be completeness, and on that test the account now does something it could not do before. It says why there are three reflections and not two or five; why there are four elements and not three; why the turn erects what it erects rather than something else; why the twelve sectors pair into six and not otherwise; why observable quantities square; why position and momentum will not both come sharp; why the massless

case sits on the boundary; why the closing step needs no carrier; why the ticking of partitions is not time; and why there is a second tier at all. None of those was put in by hand. Each falls out of the ordering, the modality cycle, or the algebra. Whether they are all correct is another question, and several of them are checkable, which is the first time that has been true of this part of the scheme.

Part VI — Positioning

The theory now says enough to be argued with, and it is better to meet the standard objections here than to have a referee raise them first. What follows is the short form of each; each deserves its own positioning note in due course.

The rival mathematical ontologies

Virtualism is not the only account that makes reality abstract, structural or numerical at the bottom, and it will be filed under one of the existing labels unless it says plainly where it stands. So here is the field, with my position on each. The comparison is more useful than it first looks, because the rivals fail in the same place, and it is the place this account was built at.

Tegmark's Level IV

The Mathematical Universe Hypothesis holds that the physical world is not merely described by mathematics but *is* a mathematical structure, and the Level IV multiverse is what follows: every mathematically consistent structure exists physically as a universe. There is no privileged mathematics — if a structure can be specified without contradiction, it is out there. Ours has the laws it has because all possible laws are realised somewhere. We are, in the phrase, self-aware substructures inside a giant mathematical object. And there can be no Level V, since any alternative would itself be a mathematical structure.

On the face of it that agrees with everything above. It does not, and the difference needs stating exactly, because a referee who files this under MUH will have missed the point.

Tegmark's Platonism is static. All the structures exist, timelessly and equally; the question of why there is anything does not arise, because the structures were always there. Virtualism's number is dynamic. It is not a pre-existing library but a production line, churned out by paradox, unbounded rather than complete. Existence bootstraps itself, and the bootstrapping is the answer to why there is anything.

That is what disposes of Level IV. If mathematical structures subsist, every consistent one is a universe and we are drowning in them. But number does not subsist. It is produced, in order, one chip at a time, by the resolution of paradox against what has already been produced. There is one production line because there is one paradox of nothingness to start it, and every emergent novelty has to be built on the number line that line produces. An Existence MkII would need another void, another beginning, and another set of numbers. I see no room for one. The actual Universe with its history of facts is plenty.

Note what Tegmark has to do about the rest of his ensemble. Most mathematical structures cannot support stable matter or complexity, so most of Level IV is a desert, and observers are confined to a thin band of Goldilocks structures. That is an anthropic move, and it is load-

bearing: with everything realised, nothing about the ensemble explains why we find ourselves in a structure that permits us, so the selection has to be done by the observer's own existence. Virtualism needs no such move, because it never populated the desert. One line, forced at every step, with no unrealised alternatives to select among.

The “no Level V” argument is right on its own terms and misses this account. It holds that any rival must itself be a mathematical structure and therefore already a member. Quite so — if structures subsist. Virtualism is not a Level V. It is the denial that the ensemble is populated at all.

Ontic Structural Realism — the nearest rival

Ladyman and Ross argue in *Every Thing Must Go* that there are no things, only structure. Relations are ontologically primitive; what we call particles are intersecting points where relationships meet; the search for a fundamental brick of intrinsic matter is a hangover from having evolved to track rocks and fruit. Their evidence is quantum: entangled particles lose their individual identities and only the holistic wave function is left, and in quantum field theory a particle is a temporary localised excitation with no persistent stuff behind it. They distinguish physical structure from bare mathematics by giving it modal character — structure carrying causal necessity, the structure of what must, can and cannot happen.

This is much closer to Virtualism than Tegmark is, and it is the position a referee is most likely to reach for. Almost everything OSR asserts, Virtualism asserts too: relations before relata, no bottom brick, the wave function as what is really there, the categorial distinctions as facts about structure rather than kinds of substance. If the account needs a genus, structural realism is it. Benacerraf's structuralism about number, treated in Part I, is the same position taken about arithmetic rather than about physics, and it fails in the same place and for the same reason.

Two differences, and both favour this account.

OSR cannot answer the relata objection, and Virtualism does not have to. The standard complaint — Heil's version is the sharpest — is that “relations without relata” is not a coherent position. A relation needs terms, as a web needs threads and a smile needs a face. OSR's reply is that quantum mechanics has broken the old definitions and our logic must catch up, which is a promissory note rather than an argument.

Virtualism does not owe that note, because it does not claim there are no relata. It claims relata are *downstream*. A relationship between two nodes is itself a whole thing with a single identity, which makes it a node, available to enter into further relationships. The levels stack: pairs of relationships have relationships, collections of them form structures, structures relate to structures. So relations are primitive and relata are real, and the second is produced by the first. The smile does have a face; the face is what the smiling built. This is foundations_v003.md §2, and it was not devised to meet the objection, which is the best thing about it.

It is also what makes tiers possible at all. If relations were not themselves available as relata, nothing could be built on the closure of a loop, and the architecture would stop at fourteen steps.

OSR is a reading and Virtualism is a derivation. OSR arrives at structure by looking at completed physics and noticing what has dropped out of it. That is respectable, and it is also

why the position has no account of where the structure came from, why there is this structure rather than another, or how one part of it gives rise to the next. It reads the answer off the back of the book. Virtualism starts from a paradox that cannot be declined and shows the structure being forced, step by step, with each step's modality fixed by what the last one ran out of. Whether the derivation is correct is a separate question. But a derivation and a reading are different kinds of thing, and only one of them can be wrong in an interesting way.

The modal-structure move deserves one more remark. OSR says physical structure differs from mathematical structure by carrying causal necessity — what must and cannot happen. That is right, and it names the thing without explaining it. On this account the necessity is paradox: what must happen is what a contradiction leaves as its only remaining move.

Wolfram's Ruliad

Wolfram's project is computational rather than mathematical. The universe is generated by simple discrete rules executing on a network of relations; space and time are not fundamental but emerge from the graph; and the Ruliad is the entangled limit of all possible computational rules. We see orderly laws because we are computationally bounded observers sampling one coherent slice of it.

Wolfram is nearer than Tegmark on one axis, because he has process. Execution is not subsistence, and a universe being computed is at least doing something. But the Ruliad makes Tegmark's move one level up. Taking *all possible rules* to be co-present puts him back in the position of having everything realised and needing something else to explain why we see what we see — and the something else is the observer's cognitive limits. That is a heavy burden to hand to psychology. The objection usually made is that it turns physics into an account of why human brains perceive order, when what wanted explaining was the order.

Virtualism has no ensemble of rules and so needs no observer to pick out a slice. There is one sequence, and at each step only one move is available.

The other standard objection to the Ruliad is the hardware paradox: a program cannot run without something to run on, and the Ruliad never says what the substrate is. This is where the two accounts genuinely part company, and it is worth being clear that Virtualism does not have the problem rather than that it solves it. A computation needs something to compute with. A paradox needs only to be a paradox. There is no program here and nothing executing it — only facts, which are relationships, and the impossibility of a contradiction standing.

There is also the question of where Wolfram's rules come from. They are given, and a theory of everything that begins by helping itself to a rule set has not begun at the beginning. The nearest thing to a rule in this account is that paradox demands resolution, and that is not an assumption bolted on. It is what a contradiction is.

Two further criticisms are worth knowing, because they will be levelled here too. That the Ruliad explains everything and therefore nothing, being unfalsifiable. And that it repackages work already done — causal sets, loop quantum gravity — without engaging with the difficulties those fields have already found. Both are fair warnings for any outsider's cosmology, this one included, which is why Part VII keeps a list of what is checkable and Appendix B keeps a note on prior art.

Smolin, and whether time is fundamental

Smolin is the odd one out, and the most congenial. He rejects the timeless mathematical reality that Tegmark and the string theorists share, calling it an intellectual trap. Time is real, the present moment is privileged, and the laws of physics themselves evolve. His relationalism is explicitly Leibnizian: space and matter are not absolute coordinates, and a particle is defined by its relations to everything else. He also proposes cosmological natural selection, with universes reproducing inside black holes and the constants adapting.

The agreements run deep. Relations rather than absolutes — the same Leibniz, read the same way. The present privileged: only Now is real, the past being true but not real. And laws that are not eternally given: on this account a law is the seventh-dimensional fact the loop deposits, which is a considerably stronger version of Smolin's claim than he makes, since it says not only that laws emerged but at which step and by what transform.

The disagreement is over time, and it is narrower than it looks. Smolin makes time the one fundamental thing. Virtualism makes it emergent, on the argument in Part V: time requires parallel processes acting as clocks to one another, which requires spatial difference, which does not exist before Earth. But what Smolin is really defending is that *becoming is real* and the block universe is false, and on that we agree entirely. Where I differ is over where the reality of becoming lives. He puts it in time. I put it in paradox, and take time to be what becoming looks like once there is space to measure it in. The numerogenesis production line is becoming without time — a sequence, and a sequence is not yet a clock.

Cosmological natural selection I have no room for. Universes reproducing needs an ensemble of them, and that is Existence MkII by another route.

Wheeler's "It from Bit"

Wheeler held that physical objects are not fundamental, that reality is built out of binary information choices, and that the universe is participatory — activated by the questions observers put to it.

He is right that the "it" comes from something non-substantial, and wrong twice over about what.

A bit is a difference, and a difference between *what*? On the foundations a relation needs two nodes with partial sameness as well as partial difference: without sameness there is nothing to relate, without difference nothing to discern. A bit gives you the difference and helps itself to the sameness. Number is prior to bit — to have a binary distinction at all you already need two, and two is the first partition of unity. Wheeler's bit is step 2 of this account mistaken for the whole of it.

The participatory half is the worse problem. Making observers' questions constitutive puts consciousness at the ground floor, and on this account consciousness is many turns up the fractal, running over a substrate the base loop had to deposit first — and, as of v007, over a surplus that could not exist until a loop had closed. Nothing at step 2 is in a position to ask anything.

Causal Set Theory and Loop Quantum Gravity — the friendly neighbours

These are physics programmes rather than rival ontologies, and both are closer to this account than any of the philosophy is.

Causal Set Theory. Sorkin's proposal is that reality is a growing network of discrete events partially ordered by causation. Spacetime is not a container; the volume of a region is got by counting the events in it; and the universe grows by sequential accretion, new elements being born one at a time. The slogan is "**Order + Number = Geometry**".

That slogan is very nearly what the Fire quadrant does, and I had not seen it before writing this section. Order and number giving geometry is steps 2 to 4: the partitions come in order, they are counted, and reflecting the count produces line, area and volume. The agreements go further. Discreteness — continuity is the mark of the void and granularity the mark of existence. Growth one element at a time — each number carved out whole, one chip at a time. Becoming as a physical process rather than an illusion. Volume by counting rather than by measuring a continuum.

Two differences. Sorkin takes the causal order as primitive and derives geometry from it; Virtualism takes paradox as primitive and derives both, since the relation of sameness-and-difference is what a causal link is made of. And growth in CST is stochastic — the sequential growth models put a probability distribution over which element is born next — where here each partition is forced, being the resolution of the hyperdox of unity against the most recent partition. Nothing is drawn from a distribution. There is only one move.

Of everything in this survey, Causal Set Theory is the programme whose results would bear most directly on this one, and it deserves proper reading rather than the summary I have given it.

Loop Quantum Gravity. Rovelli and Smolin's programme quantises general relativity while insisting on background independence: space does not exist prior to the laws and must be generated by the theory. Space at the Planck scale is a spin network, a web of quantised lines of gravitational force whose vertices are chunks of volume and whose edges are chunks of area. Evolving the network traces out a spin foam, and time is the discrete rearrangement of the knots.

Background independence is exactly right, and it is foundations_voo3.md §4: dimensions are consequences of relationships, not a backdrop relationships sit in. Quantised volume is the same claim as "to be an object is to be whole, and to change is to move by a whole amount". Where the accounts differ is in level rather than in kind. LQG quantises space, and on this account space is already emergent — it arrives with the first turn. So LQG is a theory of the Earth quadrant's extrinsics, working one storey above the base loop. That is not a criticism. It means the two are not competing, and that if the base loop is right, something like LQG is what the storey above it ought to look like.

One connection worth chasing rather than claiming. LQG labels its spin networks with representations of $SU(2)$, which is precisely the machinery Part V wants for the double-cover calculation. Whether that is a coincidence of both accounts needing the rotation group in three dimensions, or something more, I do not know. It is on the open list.

The one objection they all share

Read the criticisms of the four philosophical accounts side by side and the same complaint appears each time in different dress.

Against Tegmark: static equations are causally inert, so the hypothesis cannot say why anything happens or why time seems to flow. Against OSR: mathematical structures are abstract and cannot push, pull or collide, so something physical must underwrite the dynamics. Against Wolfram: a program needs hardware, and no substrate is named. And behind all three, Hawking's question — what is it that breathes fire into the equations and makes a universe for them to describe?

It is one objection. Every account that makes reality abstract at the bottom is asked what makes the abstraction *go*, and none of them answers without either smuggling in a substrate or declining the question.

The answer is paradox. There are four kinds, they are the four relational types, and they are what makes anything happen at all. An equation cannot act, which is true and is why Tegmark is stuck. But a contradiction is not an equation. A contradiction is a situation with no stable configuration, and the absence of a stable configuration is not inert. It is the one thing in the whole account that could not possibly sit still.

That is the pitch, and it should be made in those terms, because it is the question the field agrees matters and the place where this account has something the others do not. Everything else in this document — the fourteen steps, the four turns, the Born rule, the tier boundary — is the working out of what happens once the fire is lit.

Four objections a philosopher will make

These came up when I put the claim that reality fundamentally *is* number in front of a devil's advocate. The claim was: I defy you to describe any single thing in a manner that denies its mathematical property of being one.

That “one” is equivocal. The objection: “one rock” uses “one” as a grammatical determiner marking a singular noun, not as a mathematical concept. A thing has unity — it is a whole, undivided thing — before anyone counts it, and counting is an afterthought. Equating linguistic singularity with the number 1 is a category mistake.

The reply concedes the premise and denies that it helps. Unity before counting is exactly my claim. What the objection calls “unity, prior to number” I call number, and the burden is on the objector to say what the difference amounts to. If a thing is whole, it is one thing; if it is one thing, the fact of its oneness is a fact about it, holding whether or not anyone counts. That fact is a virtual object, and it is the most universal one there is. The objection wants unity to be metaphysically respectable and number to be a human bookkeeping convention laid over it. But strip the bookkeeping away and unity is still there, still doing the same work, and still identical across every instance of it. Break the rock in half and you have two, whether or not anyone is there to say so. The mathematics dictates the reality of its form. There is no residue left over that the word “unity” names and the word “one” does not.

Qualia. The objection: you can measure the wavelength of light, but the experience of seeing red is not a number; a philosophy that rules out direct conscious experience because it is not mathematical has denied the starting point of human reality.

Rewritten in v007, because the old reply said something this tier is not entitled to say. The old version ran: experience is the subjective side of the loop, Q3 and Q4. That will not do, and the fault is the word. Q3 and Q4 are *internal* in the sense fixed in Part I — their relations face inward and they have no outward address — and their objects are amplitude, interference, entanglement, capacity, the light cone and the volume of admissible futures. Those are physics. To call them experiences because they share a word with experience is to claim a proto-panpsychism the theory does not need and cannot defend, and a referee would be right to say so.

What this tier is entitled to say is four things, and they are enough.

First, the objection assumes that being unmeasurable is being un-numbered, and that does not follow. Half the objects in this document are unmeasurable and every one of them is numerical. The light cone cannot be photographed and it decides what may happen everywhere.

Second, I do not reduce experience to neural firing rates, which is the materialist's reply and concedes too much. The Hard Problem is hard for a materialist because he has to get experience out of matter and matter is Q2. I do not have to, because Q2 was never the whole of it.

Third, the architecture has a place for experience and says where: not in this loop, but at the tier this loop's closure forces. The surplus that Fire cannot hold is a fact about an inside carrying no inside, and whatever experience is, it is what happens where such facts have somewhere to go. That is a location and a licence, not an explanation.

Fourth, and this is the commitment Tier 1 does make: a mental instance of a fact and a physical instance of it are instances of the same fact, because instancing is not transmission and there is no separate realm for anything to travel from. Whether the two instances are *identical* is not settled here and is on the open list. It is Iconism's question, and Tier 1 needs to be told how much of it it is on the hook for.

The map is not the territory. The objection: mathematics is a tool that works by stripping away the messy non-mathematical detail of the world; claiming reality is number because physics uses number is like claiming a city is a paper map.

The analogy is the whole of the argument, and it fails on inspection. A map is a distinct object made of distinct stuff — paper, ink — standing in a representational relation to a city made of other stuff. The objection needs the “other stuff” to be there for the analogy to work. Point at it. When physics strips away the particles what it finds is charge, spin, mass — values. There is no muddy substance underneath. If someone wants to insist that the relations described are relations *of* something, the honest question is what that something is supposed to be, over and above the relations it stands in, and no answer has ever been given that does not amount to restating the demand. Virtualism's answer is that relations are what there is, and that relata are what relations produce.

Non-mathematical truths. The objection: “cruelty is wrong” is a truth about reality, understood perfectly by humans, with no mathematical properties, dimensions or numerical values; to deny the existence of justice, meaning or love because they will not go into an equation is a narrow view.

I agree that it is a truth and deny that it is non-mathematical. Notice what the objection has actually done: it has picked truths from the far end of the fractal expansion and pointed out that they do not look like arithmetic. They would not. Neither does gravity, and gravity is number origami. A moral fact is a relationship of the same kind as every other fact, holding at a level of the architecture many turns above the base loop, where the substrate is rich enough for shared awareness to be one of the relata. The content of a particular moral fact is Tier 3's business and I do not pretend to derive it here. But the form of it is a relationship, and relationships are what number is. The objection assumes that "mathematical" means "expressible in the arithmetic of the base loop", and on that reading almost nothing in the world is mathematical, including most of physics.

Why zombies are not conceivable

A zombie is supposed to be a being physically identical to a conscious one but with no inner life. On this account that is not a coherent supposition, and the reason is structural rather than a matter of finding the notion distasteful.

The form of the argument is this. The inward, internal side of a loop is not an optional extra layered on top of the outward side. It is what the outward side's over-determination forces. When a quadrant runs out of outward targets something must give, and what gives produces the interior. A being with the full constitution of a conscious one has, by that constitution, the over-determination that forces the turn. To have the constitution and not the interior would be to have a hyperdox that Nature declined to resolve, and Nature does not decline.

Amended in v007: the argument must be run at the right storey, and up to v006 it was not. Stated of the base loop it establishes only that a physical duplicate has Q3 and Q4 in the base-loop sense — amplitude, interference, capacity, the light cone — which is to say that it has an inside in the sense in which an electron has one. True, and not the conclusion wanted. The argument delivers what it promises only when it is run at the tier where the substrate is rich enough for the interior in question to be an experiential one: duplicate the substrate, and you duplicate the surplus; duplicate the surplus, and you duplicate the forcing; duplicate the forcing, and the turn happens, because it is not the sort of thing that can be declined.

Tier 1 supplies that form and cannot supply the content, because saying what the over-determination at that tier consists in is Iconism's work. What Tier 1 does supply is the guarantee that the forcing is available at every tier, since it is the same mechanism at every boundary — which is what the tier-boundary argument in Part V establishes and what the base-loop version was borrowing without having earned.

This also means conceivability arguments are doing less work than their users think. That I can *entertain* a description of a physical duplicate without an interior shows something about descriptions and nothing about *dasein*. Words wave hopefully in the direction of situations. They are not the situations.

Part VII — Housekeeping

Changes made in v008

Listed so that nothing changes by stealth.

- **Q3 is renamed Internality and Q4 is renamed Individuality**, on the author's ruling of 21 August, overriding the cautious option taken at v007. The four kinds of Existence are now Factuality, Reality, Internality, Individuality. *Possibility* is retained as what Q4's void consists in rather than as its name, so the anodox / distant-future / light-cone / admissible-futures chain is unaffected. Applied at Part I, Part III and Part IV, with the author's note on the choice of words carried in full.
- **The naming turns out to agree with the deposit.** What step 14 sends up was named the fact of individuality at v007, from the deposit's own character and before the quadrant was renamed. The quadrant that hands the surplus upward is now the quadrant the surplus is named after. Recorded at step 14 and in Part I, since two independent roads meeting is worth more than either road.
- **A new standing note, *On the question of Reality*, is added to Appendix A.** The author's, written 21 August. It answers why Euclidean objects become relative — only perfectly 3D objects can be Euclidean, and anything foldable in four dimensions or more must be boosted in three — and carries the unit-sphere argument, the folding-against-bending distinction, the derivation of negation from rotation through half a circle, and the conclusion that the imaginary dimension is the real dimension seen from the other end.
- **The unit-sphere argument is summarised at Part II**, where the 4D hyperdoux is first asserted, with a pointer to the standing note. v007 asserted that the 4D version is smaller in radius and left it there, which invited the objection the note now forestalls.
- **Tier 2 is characterised as behaviour rather than being.** New material at the end of Part V's *What the closure hands upward*, from the standing note's closing line. Tier 1 is about what things are; Tier 2 is about what they do. Set against the tier-boundary argument the two are one observation from opposite ends: what Fire cannot hold is a fact about a doing, so the surplus and the subject matter of the tier above are the same thing. Qualified as the note qualifies it — behaviour is not absent from Tier 1, since natural mathematics is what facts do; the difference is which is the subject.
- **Whether Iconism is Q3's or runs across all four quadrants** is now an open question rather than an assumption, per the standing note. It is the first question the interface note puts to Tier 2.
- **The Mandala is described as a map of all possible combinations of relation**, at Part II, which fixes what can be criticised about it: the fundamental nature of virtual relations, and after that only the interpretation of the map.
- **Smaller.** *Unbounded is an adjective*, added where the verb-not-noun point is made. Two cautions added on the new naming, since *internality* the property belongs to both quadrants while *Internality* the quadrant is Q3, and since *subjective* remains correct of Q3 and Q4 together — it is only the slide to *experiential* that is refused.

Changes made in v007

Listed so that nothing changes by stealth.

- **The ordering is derived, not merely confirmed.** Part III now leads with the derivation from the direction of paradox — emergence runs fire → earth → air → water, a quadrant takes its targets in that direction, and it cannot take itself — and demotes the Fire text and the sign list to independent confirmations. The derivation has been in Appendix A since v002 in a single clause and should have been in the body years ago. This matters for publication: the ordering no longer rests on a chart at any point, and the sign list becomes a test that could have failed rather than a source.
- **The loop of reflection is worked out.** Appendix A's claim that the first sector of a quadrant reflects in the final sector of the next is exact, closes the circuit from Water back to Fire, and extends: the seconds pair across the diagonal, so all twelve sectors fall into six mirror pairs with none left over. New section in Part III, with the table. Cross-referenced at steps 3, 5, 6, 8, 9, 10, 11, 12 and 13.
- **What the mirror corroborates.** The pairing is forced by the ordering rule and is therefore bookkeeping, and I say so. What is not bookkeeping is that all six pairs mean something and nothing arranged them: separation against entanglement, one allowance against every admissible future, capacity against Euclidean volume, and the first area ever made against the very area the Born rule squares. It is the ordering's strongest corroboration and the only one owing nothing to the zodiac. The Born rule argument in Part V gains it as a second prop.
- **The closure deposits two things.** Step 14 gains a subsection. Law is the sameness half of Leibniz's Law arriving in Fire; the fact of individuality is the difference half, and it had gone unread. The second is the half the tier above feeds on.
- **The tier boundary is derived.** New section at the end of Part V, *What the closure hands upward*. Fire can hold law and cannot hold a fact whose content is that there was an inside. Actual, unpreventable, determinate on both sides: a dimension is forced. A tier begins where a closure deposits a fact that factuality has no room for — which is step 1 run at the top of the loop. This closes the largest hole in the document, which was that the fractal expansion had been asserted and never forced. Full argument in `tier_boundary_v002.md`; step 1 now flags itself as the template.
- **A criterion for tiers, in place of a count.** A closure depositing law forces nothing and is the ordinary fractal iteration; a closure depositing individuality forces a tier. So tiers are numbered by regime rather than by iteration, and the question whether Tier 2 is the *immediate* next tier dissolves.
- **Internality replaces the structural sense of subjective.** New subsection under *Existence*. The word had been doing two jobs — inward-facing orientation, true of Q3 and Q4 at every tier, and experiential, which is Tier 2's. The first is now *internality*, with *idiotropic* as its older cousin from step 1. Applied throughout Parts I, III, IV and VI.
- **The qualia reply is rewritten**, because under that ruling its old formulation is no longer sayable. It had said experience is the subjective side of the loop, Q3 and Q4 — which, taken at the base loop, claims that amplitude and the light cone are experiences. The replacement claims four narrower things and is stronger for it.
- **The zombie argument is relocated one storey up.** As written it established only that a physical duplicate has an inside in the sense in which an electron has one.

Run at the tier boundary it establishes what it wanted. Tier 1 supplies the form and hands the content to Iconism.

- **Sorites rather than Theseus.** Stated in Part I, where it is the general form of what the nested triangles demonstrate: the fact is not made of its truth-makers, so replacement does not raise an identity question, and what is left over is a question about where a boundary has not been drawn.
- **Leibniz's Law gains its second half explicitly**, and is named as the glue between quadrants — each supervening on the others by being identical with them in some shared fact.
- **One past, not four.** Noted in Part V under *Time*. Every quadrant contributes to the single classical past whatever its modality, and whether or not time is in the neighbourhood. What differs is the kind of entry, not the ledger.
- **A note on what the zodiac is doing**, added under *Mandala*, since v007 changes the answer: it is a test the derivation had to pass, not a premise.
- **The double-cover section gains a suggestion:** the twelve reflections pair into six mirror couples, and a mirror couple has a known relation between its normals, so the composition may be more tractable in six than in twelve. Worth half an hour before attempting the general calculation.

Corrections and changes made in v006

- **The heart of a whole is defined**, under *Virtual*, with the triangle as its worked case. A two-ended relation has a midpoint for free; a triangle's centre can be terminated on only by importing a relation the figure does not contain; the midpoint route imports nothing, runs without end, and never arrives; and every triangle in the sequence has the same centre, so the parts may be replaced for ever while the heart stays put.
- **The higher-dimensional claim is corrected and turned into a worked example.** In three dimensions the edge midpoints of a tetrahedron form an octahedron taking half the volume — not a simplex, not similar to anything in the figure, and not predicted by anything in the two-dimensional case.
- **The set-theoretic objection becomes a diagnosis**, under Leibniz's Law, by way of Extensionality; the von Neumann concession is added.
- **Benacerraf is added**, as an ally on the identification problem and an opponent only on the staticness of structure, with the 1973 dilemma diagnosed as falsidical by way of its causal theory of knowledge.
- **The claim against infinity is narrowed** to completed or actual infinity, which is Aristotle's distinction.
- **The latent-orthogonal rule is stated once**, under *Dimension*.
- **Separation as a void is defined**, under *Existence*, with the three fences distinguished.
- **Mandala gets a definition in Part I** rather than waiting for Appendix A.

- **The twelve/seven paragraph is rewritten**, since seven is load-bearing at step 14 and in the closing summary.
- **The ordering paragraph in Part III argues rather than asserts**, on why the allowance must be the line and the interference the area.
- **Two errors corrected.** *Relation* had described two indiscernibles collapsing into one; and hyperdix is **foundational in Earth**, not third.
- **Smaller repairs.** The Hegel sentence; Zermelo; QCD not QM; “twelve extrinsic dimensions”; Sherlock retired as a premise and kept as a joke.

Corrections and changes made in v005

- **Part VI’s first section is expanded** from a note on Tegmark into a survey of the rival mathematical and structural ontologies: Tegmark, Ontic Structural Realism, Wolfram’s Ruliad, Smolin, Wheeler, and Causal Set Theory and Loop Quantum Gravity as neighbours rather than rivals. The section closes on the objection common to all of them.
- Nothing else in the document was altered at v005.

Corrections and changes made in v004

- **The ordering ruling is closed**, on the independent evidence of the sign list in `mandala_framework_v005.md` §8. v003 had it ruled but flagged. (v007 demotes both witnesses beneath the derivation.)
- **The involution’s bookkeeping is ruled.** It stands outside the four powers of i , because a rotation needs a plane and at step 1 there is none.
- **The loop is told once.** v003’s “Transforms and the Mandala” narrative has been distributed: its mathematics into Part II, its overview of the second half into Part III, and its step-specific content into the relevant steps of Part IV.
- **The $4 = 1 \times 4 = 2 \times 2$ paradox is restored** at step 3, where v003 had lost it.
- **Time gets a section**, and the question Fire left hanging is answered with a principle.
- **Part VI is new.** The positioning against Tegmark and the four objections were previously only in conversation.
- **Part V gains the double-cover section**, marked as new and as reconstruction.
- **The retrieved reference material is parked** in Appendix B rather than left in the body or discarded.

Carried unchanged from v003: the fourteen-step spine and its numbering; the correction of the within-quadrant ladder; the identification of amplitude, phase, entanglement, capacity and the light cone; the Born rule argument and its modulus-square repair; the citation of foundations §3 rather than §2; cube roots rather than square roots at step 4.

Where this goes beyond the record

Marked so that what is mine and what is reconstruction stay separable.

On the record and used as given. The four powers of *i* and their forces; the elemental order; boost and un-boost; the wave function as the emergent fact of wholeness; the seventh dimension as a thing becoming true; the fractal reading of the higher loops; the counter-rotation of inward and outward; the modality cycle orthodox → hyperdox → hypodox → anodox; the Fire quadrant entire; the definitions of Part I, including the author's own formulations of the heart, of separation as a void, of the latent orthogonal, and of the Mandala's naming; the whole of Part III's account of the second half; the claim that a continuous loop of reflection surrounds and is within the loop of rotation; the positioning of Part VI against Tegmark and the four objections. The summaries of the rival positions in Part VI — Tegmark's ensemble, OSR, the Ruliad, Smolin, Wheeler, CST and LQG, together with the standard objections to each — are retrieved rather than mine or the author's, and should be checked against the primary literature before anything built on them goes into print.

Reconstructed by extension, consistent with the record but not stated in it. That the three reflections take the non-self elements in wheel order beginning with the successor, and that the break is the denied fourth returning to the successor. That the turn erects, intrinsically, what the denied reflection was reaching for extrinsically. That the twelve extrinsic, the four intrinsic and the seven erected are counts of different things. That each turn opens its successor's line, with Fire the explained exception.

Reconstructed for the Earth quadrant, where the original had blank bullets. 4De as separation and relative position; 5De as the constraining of volume, the Machian rung; 6De as joint classical repositioning, over-determined and hence the break's cause. The spine is the author's own closing summary — “the transfer of particles, the constraining of volume, classical repositioning” — but the prose is reconstruction and should be read as a proposal.

New identifications, offered as candidates. Amplitude as the line of Air and phase as its area, with the Born rule falling out as dimensional bookkeeping; entanglement as the over-determined volume of Air and therefore the cause of the third break; capacity as the line of Water; the light cone as the area of Water, with *c* read as a shape rather than a speed; the volume of admissible futures as the terminal paradox that forces the closure.

New in v004 and mine. The double-cover question and the reading of the vanishing sum as net displacement against phase. The three consequences drawn from the time principle. The framing of the four objections' replies, though the positions replied with are the author's.

New in v005 and mine. In the survey of rivals: that Virtualism escapes the relations-without-relata objection because relata are downstream of relations rather than absent; that Tegmark's Goldilocks-band argument is an anthropic move made necessary by his ensemble; that the Ruliad reproduces Tegmark's move one level up; that Smolin's disagreement about time is narrower than it looks; that Wheeler's bit helps itself to the sameness a relation needs; that LQG works one storey above the base loop; and that the four accounts fail at one joint, which is the joint this one was built at.

New in v006 and mine. That the heart of a two-ended relation costs nothing while the heart of a triangle cannot be terminated on without importing a relation. That the octahedron in the middle of a subdivided tetrahedron is a worked instance of emergence forced by a dimension. That the Extensionality objection is Leibniz's Law applied to the construction. That Benacerraf's identification argument is an ally and his 1973 dilemma falsidical by way of a causal theory of knowledge.

New in v007 and mine. That the ordering follows from the direction of paradox together with the prohibition on self-reflection, and is therefore derived rather than confirmed — the clause is the author's, in Appendix A; promoting it to the argument and noticing what it does to the zodiac objection is mine. The working-out of the loop of reflection into six exhaustive mirror pairs, the adjacent/diagonal pattern, and the reading of the six as corroboration of the ordering and of the Born rule. The suggestion that the six couples may make the double-cover composition tractable. The tier-boundary argument in Part V and the criterion of law-versus-individuality that follows from it — the observation that the deposited fact does not fit in Tier 1 of Q1 is the author's, from the bridging note of 20 August; recognising it as a hyperdox meeting all three conditions, and drawing the criterion out of it, is mine. The internality/subjective separation and the consequent rewriting of the qualia reply and relocation of the zombie argument.

New in v008. The naming of Q3 as Internality and Q4 as Individuality is the author's ruling, and the note on the choice of words is his. *On the question of Reality* in Appendix A is his entire, written 21 August, and is apparatus rather than draft: the unit-sphere argument, the folding-against-bending distinction, the derivation of negation, the peaks at 3D, 5D and 7D, the holographic reading of Q3, the Ryle-and-Koestler reading of Q4, and the characterisation of Tier 2 as behaviour are all his. Mine at v008 is only the joinery: noticing that the renaming agrees with what the closure was already said to deposit, and that the behaviour characterisation and the tier-boundary argument are the same claim approached from opposite ends. Both are cheap observations and neither adds a premise.

The Born rule identification remains the one to push hardest, framed as a derivation of the inner product rather than of Born-given-the-inner-product. The light cone identification remains the one to be most careful with, though it is better defended at v007 than it was, since it now has a mirror partner in the Machian rung.

What remains open

- **The double-cover calculation.** Does the base loop land at +1 or -1? Decidable, and still the most promising loose end. Try the six mirror couples before the twelve singletons.
- **The turn-erects-what-was-denied claim.** It works in all four cases, which is either strong evidence or a sign that it was built to. Worth an independent test.
- **The Earth reconstruction.** Steps 6 and 7 are the least-supported prose here.
- **Probability proper.** Positive, quadratic and normalised is not yet a frequency. The tolerance account is a position, not a derivation.
- **Whether number production is linked to causation.** Two candidate answers, both consistent with everything above, no way yet to choose.
- **The medial recursion above three dimensions.** What sits in the middle of a subdivided 5-cell I do not know, and whether the sequence of middles is itself structured is a question with an answer.
- **Whether the heart is a general principle or four separate cases.** Asserted generally; demonstrated for a relation and for a triangle.
- **The two off-diagonal generator cells** from loop_closure_synthesis_v001 §9.

- **Whether the ticking of partitions is Planck time.**
- **Acceleration at Water.** Flagged provisional in mandala_framework_v005 §9 and not developed here. It belongs with gravity.
- **The qualia reply, and with it Benacerraf's second dilemma.** Properly Iconism's. Virtualism is committed to a mental instance and a physical instance being instances of the same fact; whether they are *identical* is Tier 2's. Tier 1 needs to know how much of it it is on the hook for. Handed over formally in iconism_interface_v002.md §9.
- **The void and ignorance.** Whether the intrinsic void of individuality is merely analogous to ignorance or identical with it. Iconism's, and now handed over with a precise object to test it against: a void fully determined by what borders it, occupied by nothing, invariant under the replacement of everything that determines it. If ignorance has that shape the identity claim is available; if not, the analogy has a limit that should be stated.
- **Whether LQG's SU(2) labelling connects to the double cover.**
- **Causal Set Theory, read properly.** The nearest programme in mainstream physics, and I have it at second hand.

New at v008:

- **Is Iconism Q3's, or does it run across all four quadrants?** The standing note raises it and declines to settle it. If Tier 2 is the behaviour of Tier 1, and all four quadrants have behaviour, the second answer is the natural one — but that is an argument from tidiness and wants better. First question to Tier 2.
- **The peaks at 5D and 7D.** *On the question of Reality* records that 3D is one kind of peak and that there are others at five and seven dimensions, peaks of surface area and of volume. The document has a seven that is load-bearing and a 5De that is the Machian rung, and it would be careless not to ask whether these are the same sevens and fives or a coincidence of two unrelated countings. Probably the latter; worth an hour to be sure, because if it is the former it is a considerable result and if it is not, someone will raise it anyway.
- **Whether the holographic reading of Q3 is a metaphor or a claim.** The note says Q3 thrives only on what exists at its boundary. If that is meant strictly it is a strong constraint on Iconism and possibly a testable one. If it is an image it should be marked as one.

New at v007:

- **Does the surplus have a magnitude?** The sharp version of the tier criterion treats individuality as a kind, so one instance suffices and the boundary is crossed at the first closure of that sort. A softer version has the surplus accumulating until it reaches over-determination, which needs a threshold, and thresholds want explaining. Running with the sharp version until something breaks.

- **Where do the loops between the base loop and Tier 2 run?** Chemistry and biology as fractal iterations is the obvious sketch, and it is a sketch. Nothing above requires knowing and nothing above forbids finding out.
- **Does Tier 2's loop pair into six mirrors as this one does?** It should, since the pairing follows from the ordering alone. It is the cleanest single prediction this tier makes about the tier above, and Iconism has been asked for it.
- **Is Tier 2 a counter-rotation, and if so of what?** The author's instinct is that it runs the other way. Since the drawing direction is conventional, the substantive form of the claim is that Tier 1's intrinsic direction becomes Tier 2's extrinsic one — what was a turn below is a reflection above. Recorded as the right shape of the question; whether it is true is Tier 2's to find out.

Closed since v003: the ordering ruling; the involution's bookkeeping; whether the ticking of partitions is Time. Closed at v006: the position of hyperdox in Earth; whether the void shrinks with dimension. Closed at v007: whether the ordering depends on the zodiac (it does not); whether there is a connection running from Q1 back to Q4 (there is, and it is the sixth mirror pair); what forces the tier above.

Appendix A — the author's standing notes

Carried from v002 unaltered. These are apparatus rather than draft, and nothing above supersedes them. Two clauses in them have been promoted into the body at v007 — the direction of paradox fixing the reflection order, and the continuous loop of reflection — and they remain here in their original form.

A note on the meaning of inner and outer

Inner and outer are relative concepts, rather like the world that has one hemisphere as land and the other as sea; there is no telling which surrounds which. When we consider the void and being, we can see that all of Existence emerges from a factual nothingness, and in that initial state there is a change that has to be, that forms an eternal relationship between void and being, between null and whole, between zero and unity. In this relationship there is no before and after, just being and non-being. There is no sensible way to say whether being exists within non-being, or the other way around, and together they form the whole scope of Existence; but at the same time there is no way to say whether any other thing that exists does so within that relationship or outside of it. Not even the integers — because while each integer is a series of partitions of being whole, each partition is itself whole, making the fundamental paradox of Existence be that all things are inside and outside.

Such being is not and never can be static. Paradox births Existence as facts that are unchangeable yet in a state of perpetual change.

A note on Paradox

Quine takes paradox to be the case where a proposition purportedly establishes a seemingly absurd conclusion; and further, that some will be veridical, some falsidical, and some antinomies. Into this latter group he includes Russell's famous barber and Gödel's

Incompleteness. Quine also discusses Grelling's paradox, which arises from the query: is the adjective "heterological" an autological or a heterological one?

The assumption in all the self-referential so-called paradoxes is that anything can indeed be self-referential in any more than a hand-waving manner. Grelling's paradox is a perfect example, and if we take any of its words as words, rather than as the meaningful facts pointed to by the words, then we see that not only can words tell lies, but that words can never *be* the truth — they can only perhaps tell it.

Autological, according to Grelling (1908), is a description of a word that is true of itself. One example Quine gives is "short" — yet is "short" truly short, after all "long" is shorter! The point is that words are never truths; at best they can wave hopefully in the general direction of some situation or thing, and give a hint at truth.

Grelling's paradox is all bollox, because "syllable" is in no wise like a syllable, yet we are asked to believe that "multi-syllabic" is like many syllables. This makes no sense at all. The point is that short is not short when compared with long, so Grelling's argument is in tatters.

I define various paradoxical states as not only descriptive of ontological states, but as the natures of the only four possible relational types. The methods of emergence, the engines, are all paradoxical in the usual sense; but to distinguish between them we need to introduce hypodox (following Peter Eldridge-Smith), and then hyperdodox, as contrasts to orthodox, and then to refer to a defined void I would coin the term anodox — and conclude that dialetheia abounds in all of these in one way or another. Note that we are not considering propositions and language but factuality and reality, so veridical and falsidical and antinomy do not apply.

The orthodox has dialetheia present in its fundamental nature. The number line may be the first emergent, and proceed extrinsically, but it also proceeds intrinsically, giving rationals for every interval, and then all the other number objects. But in all respects, as emergent, the orthodox is never complete; were it to be complete, then Existence would be static, something more like Einstein's vision.

The orthodox naturally gives rise to the hyperdodox of reality, with inbuilt tension eventually leading to gravity. That tension is the dialetheia. Hyperdodox separates, which leads to hypodox, and hypodox leads to quantum change. The whole cycle of change and causation is an alternating Machian/quantum sequence of consequences that paradoxically have no separation, yet cannot occur out of sequence, leading to the emergence of time. Note: Time is not the past, but the past is ordered in accordance with the chronological sequence of events. On this point SR is incorrectly interpreted, the interpreters having been seduced by the apparent reality of space and time.

Hypodox necessarily defines the anodox that is the very thing that allows hypodox to be and to change. Together they form another dialetheia, that compels anything to be both itself and that which it is not — i.e. all things must have a complement in what they are not. The complement may be a true void, or it may match some extant fact or facts, in which case those facts virtually shape the void.

On the nature of reflections

A reflection is an extension of that which is reflected, along the whole dimensionality of that which is reflected; but this means that when you reflect a 3D object each constituent

dimension must also be reflected, so the lines, the areas and the volumes are all rotated with the strong emergence of boosted space.

Within each quadrant the nature is different from the other quadrants, therefore the starting point for the reflection is different for each quadrant. The point where an unsustainable quantity of reflections occurs is always after three reflections, but what is being reflected at the final stage is always a unique way of forcing the next quadrant to emerge, matching the element of the next quadrant. It will be apparent here that “final” is an ambiguous term in regard to the maze of the Mandala, because there are two directions of rotation, the intrinsic and the extrinsic, and these run in opposite directions. But it should be realised that all regions overlay one another, so what is important is that the order is maintained, and the order is the same causal order of fire, earth, air, water — i.e. the direction of paradox, where any inherent paradox demands resolution within nature.

The orthodoxy of numbers leads to 1D hyperdogy — the final integer partition always being one more than the quantity of integers in Existence*, producing the number line by repeatedly reflecting the point of being; reflecting the line is 2D hypodoxy, because the numbers can fold through possibility, the natural consequence of the separation of hyperdogy; hypodoxy reflects again because it can, defining, causing or creating the anodoxy of volume, i.e. 3D space.

- incidentally, this places the cutting edge at the opposite end of the spectrum from Cantor’s diagonal argument.

Having reflected three times, a fourth results in hyperdogy, matching the extrinsic hyperdogy of the number line with an intrinsic hyperdogy of the earth quadrant, Q2, i.e. boosted space, aka reality. This is both a reflection of the factuality of fire and a surrounding of the first quadrant, which sees the first sector of Q1 reflected in the final sector of Q2 and so on throughout the quadrants, creating a continuous loop of reflection that surrounds and is within the continuous loop of rotation. This models the mathematical possibilities of physical symmetries, defining exactly what Nature will allow.

The challenge for understanding what then can be is to correctly extend the rules of causation, which are the rules of paradox, to extrapolate into the hidden regions of Existence by using the hidden regions of the Mandala as a guide to what is possible.

From the above we can see that hyperdogy will occur at different relative positions within each quadrant, but what is important is the full circle of paradox and emergence. The difference between each quadrant is really no more than recognising that the extrinsic and intrinsic turns can occur anywhere as long as they occur in sequence. So the Mandala, being a map of all legal steps, must include every possible change, and this is the feature that makes it such a useful guide.

On the use of elements and signs

Many may balk at the idea of naming stages of emergence using the labels of astrology, due to the bias of sceptical materialists against that ancient school of thought; yet just as chemistry was built on the ruins of alchemy, and astronomy on the remains of astrology, acknowledging the roots of disciplines cannot be taboo. Furthermore, within QCD the theory of colour force adopts the RGB of actual colours and eye biology to conveniently label the triple charge of the

strong force. For emergence there is a requirement to label the four-way and twelve-way relational nature of the unfolding fractal mapped as the Mandala.

Others may take pause due to the directions of rotation that occur in the Mandala, and are then adopted in seeming conflict with the mathematical convention from Descartes for aligning the x-axis from left to right when positive. But just as alchemy and astrology were entry levels for sciences, astrology was also the inspiration for the Mandala, and the regions depicted are for the purposes of emergence entirely abstract, except for their relations to one another. Hence my decision to align with the convention for astrological charts makes more sense than sticking to Cartesian coordinates. My choice is to use the Zodiac in the same manner as the physicist uses RGB — re-purposing a readymade in a manner of which Duchamp would be proud.

A directive on Tiers

The Mandala maps the relationships between kinds of virtual relationship, which allows an observer to see what goes where. However, the use of “where”, and perhaps even “what”, in everyday English would usually apply to real things and real places. That is not the intention with the Mandala; the what and the where are things defined by virtual relationships and so have no being or place other than that which they define themselves.

The definition of the four quadrants is that they are the only four basic kinds of relationship and ontological status, and that each quadrant emerges as the result of hyperdox, a kind of paradox of over-determination that Nature resolves by the birth of imaginary dimensions that form after the number line, the complex plane, and volume. Instead of forming extrinsically as reflections of a line, a plane and so on, quadrants form intrinsically — which is how the number line partitions, by repeatedly resolving the hyperdox of being; but the resolution of hyperdox within the Euclidean first quadrant is not a reflection of a whole line or plane, but a rotation of volume as an intrinsic Lorentz transform, as distinct from an extrinsic relation of volume to hypervolume by reflection.

The next occasion that encounters hyperdox resolves in the same manner by another such rotation giving quadrant 3; but due to the nature of reality in quadrant 2, the emergent results are entirely novel, i.e. unpredictable in terms of prior quadrants, just as the relative space of reality was unpredictable by any property of quadrant 1. So it is that each quadrant forms an entirely novel state of existence, emergent from the prior quadrant, but with novel behaviours and possibilities.

Once four quadrants have emerged, the fourth power of i simply adds the facts of quadrant 4 to quadrant 1. These facts are objective wholes, devoid of any parts, and so present a new tier for the realm of factuality — a tier where the differences that emerge anywhere can be accommodated as true facts, even though the facts themselves may be in as much conflict as when they were filled with the parts of reality, parts that reality conserves as it recycles all energy by its physical behaviour.

I do not know whether there is a theoretical limit to the number of tiers that are possible, but it is clear that Tier 1 of Quadrant 1 gains a Tier 0 of its own, due to the emergence there of roots to the number line that are no more than inversions of the extrinsic powers of the number line that give area, volume, and so on.

It is also clear that there is a world of possibility to be explored with the aid of this Golden Compass, or alethiometer as Philip Pullman might have called it. I cannot diagnose all of Existence in one fell swoop, but I can recognise that this points the way.

The four quadrants

Quadrant 1, fire, is a relation of sameness — of absolutes with no relative differences. Numbers are numbers, Euclidean objects are Euclidean objects, facts are facts, and truth is the identity of facts. It is not that this region cannot tolerate difference; rather that difference forms an absolute truth here, because there is no real space. This makes the region one of objective wholeness, from which all derived objective parts must be constrained and governed.

Quadrant 2, earth, is a relation of difference — of absolutes with relative differences. Masses are mathematical objects that possess boosted differences due to the paradox of hyperdix, and what emerges in quadrant 2 is the reality of space: 3D space with boost, which is Einstein's mass/energy equivalence with no involvement of time yet. The extrinsic and intrinsic dimensions of this quadrant give the basis of all physical reality and the emergent possibilities therein. This makes the region one of objective differences, the real parts that can form real wholes.

Quadrant 3, air, is a relation of inner difference — of subjective differences. Icons are the inner consequences of the previous quadrant: when quadrant 2 finds itself in a situation of hyperdix, something must give, and the very nature of the quadrant is that boost is hyperdix; but all dimensions define their orthogonal, and so it is with quadrant 3 emerging from quadrant 2. The complex-plane rotation is the square of i , the -1 of a square object, an area; so at its fundamental level quadrant 3 is what you get with the subtraction of something that must be accounted for and so must be added elsewhere.

We should note that material subtraction and addition — quantum change — occurs within quadrant 2. Quadrant 3 is the quadrant of light's consequences, which makes it the quadrant of what electromagnetic change does beyond the material, and I am not qualified to say what else from a physics perspective. However, it is obvious that the transfer of value across space must convey information: the thing transferred has its own data, while multiple such transfers must transfer the information inherent in the relations of their sources. Thus quadrant 3 is the quadrant of the inner space of space. For instance, transfers from three different points in quadrant 2 space that combine in a quadrant 3 sense must convey information of a triangular nature. However, the requirement of combination at the point of addition, the destination, makes this quadrant subjective, though composed of parts like the reality it reflects. This makes the region one of subjective difference.

NB. There is perhaps a distinction to be made between the transfer of energy, which would be considered material, and the transfer of information, which would be considered immaterial. Information has no mass equivalence and so does not belong in quadrant 2, though there must be a quadrant 2 cause of the transfer of information — i.e. electromagnetism per se belongs to some Q2 extrinsic, whereas the emergent consequences that are not accommodated by Q2 belong in Q3.

Quadrant 4, water, is a relation of inner wholeness — of subjective wholeness, formed by the combinations of changing information in Q3. In one sense elemental water reflects elemental air; they are two sides of a coin, but the flip from one side to the other is a rotation

in the same way that the rotation of i goes from any quadrant to the next. One way of looking at it is as a subtraction of whatever is produced in going from Q1 to Q2 — literally the subtraction of spatial difference; but Q3 space is the inner space of information, and the differences are the subjective parts. Removing the parts leaves just the fact of the whole to populate Q4, a relation that as an emergent step is better described as the formation of the subjective whole from the subjective parts. Q4 is the arrangement of the subjective parts, and that defines both what something is and what it is not.

Being a whole, an object of Q4 is factual, and it is simply orthodox for this fact to add to the totality of facts by making the final rotation from Q4 to Q1. Getting into Q4 from Q3 is the hardest thing to imagine, and as a change is similar to physical decay; whereas the step from Q4 to Q1 is a straightforward addition of fact, albeit as an inversion like a photographic negative, the water element being the space of what is not, as defined by the silhouette of what is.

*(voo8 note, not an alteration: where these standing notes say “subjective” of Q3 and Q4 they mean what the body of the document now calls **internality** — inward-facing orientation, no outward address. The notes predate that distinction, and predate the naming of Q3 as Internality and Q4 as Individuality, and are left as written on both counts.)*

On the question of Reality

New standing note, 21 August 2026. Apparatus rather than draft, like the rest of Appendix A, but written after the body above rather than before it. Where it bears on the body — the unit sphere at Part II, the naming of Q3 and Q4 at Part I, and the character of the tier above at Part V — the body now points here.

What is it that makes facts real?

This question means why is it that Euclidean objects cease being universal and become relative? Why is it, or how is it, that objects gain boost and space becomes distorted?

The answer to all this is that only perfectly 3D objects can be Euclidean, but objects that can be folded in four or more dimensions must become boosted in 3D.

Because this is quite a claim we need to dissect it to show that it is true.

The nature of the unit sphere, where in 3D the radius 1 equates to $x^2 + y^2 + z^2$, while in 4D we see $1 = w^2 + x^2 + y^2 + z^2$. The 4D unit sphere differs in that it has a novel region where the radius and diameter shrink to $\frac{1}{2}$ that of the 3D equivalent, while retaining the full 3D structure four times over.

The virtual argument is that this fact produces an apparently paradoxical situation whereby there can exist multiple values for the radius, depending on the relative perspective. This is what is meant by a state of hyperdox — multiple truths co-existing.

When a number folds, the line takes on multiple dimensions. The simple case is that of area when 4, that is 1×4 , can fold to become 2×2 . This is a case of extrinsic emergence from the number line, and it produces a novel kind of thing, area.

But there is another possibility for emerging numbers, that results from only half the function that produces area as square number, and that half step gives a complex number instead. 4 can be folded to be $1+3$, or $2+2$, or $3+1$, although convention has it that we separate the two parts by naming one part imaginary, though Nature has no need to do so. We'd write $1+3i$, or $2+2i$, or $3+1i$.

What this maths means is that there are two ways to derive area when you start with just a line of values. One way is extrinsic and reflects a number away from the line, filling out the potential novel dimension, giving square objects that are different from the linear numbers, but are still made from linear numbers.

The second way to make area is with the complex objects, and what the complex objects do by bending the linear number instead of folding it, is rotate it around a point, creating a similar area using the novel dimension, but this time the object is circular, not square, defining a circle that has no precise value other than in terms of itself — i.e. π starts off life as a crude approximation in terms of linear numbers, but is perfectly complete in terms of circles. This, I'd suggest, is rotation turning inward into area as an intrinsic emergence, and not just allowing the emergence of circles, but also the process of negation, which although we can see it from our perspective on the line of positive numbers, there is nothing about the differences between positives that forces negatives to exist, whereas rotation through half a circle does just that, forcing the emergence of negatives.

I have been attempting to show all this in such detail for the 2D Euclidean plane, so that we can easily extrapolate the story for the 4D cousins. We don't need to spell out each step of the way, but what is obvious is that the process of emergence, once acknowledged, explains the existence of lines, areas, volumes, and hyper-volumes. Also, that emergence explains the difference between extrinsic emergence, as reflection and square objects, and intrinsic emergence, as rotation and round objects. The former giving external spaces, and the latter internal spaces, but with the proviso that the spaces are the same spaces, but from different relative perspectives.

So, when we consider the hyper-rotation that is involution, when 3D objects turn inside, and then inside-out, and then back again, after the fashion of a spinor, we can see that the imaginary dimension is only the real dimension looked at from the other end.

So, when we examine the hyperdiox of 4D objects, we see that Reality is not ontologically different from Factuality. At the fundamental level, everything is composed of factual relations, that at one level look like a line of numbers, then an area, a volume, and so on. It is the nature of the numbers that makes 3D one kind of peak, but we see other peaks at 5D and 7D. Peaks of surface area and volume. What comes with rotation is negation, aka subtraction, and we ought to be able to see that both turning and subtracting are behavioural. More dimensions confer different behaviour rather than different ontology. But that behaviour can be creative, so for instance turning a line of numbers around creates a mirror image line of negatives. We could perhaps jump to the conclusion that rotation and reflection were the same for a line, but this is not so — reflection within the line is what is creating integers; it takes a novel action to break out of that and create negative integers.

As for what makes Reality real, it is the behaviour that follows from involution. But behaviour is change, and the type of change that is possible is different depending on how many dimensions are involved.

I think that properly speaking the emergence of Tier 1 is the story of different kinds of facts and spaces emerging, while what comes next is a story of change, although we are already seeing change emerge in Q1 as basic mathematic, perhaps what we should call natural mathematics — partition, addition, subtraction, multiplication, division, rotation. These are all what facts can do, at least when they are numbers. In doing so they emerge their own ontology of being, one that is both growing and static, and which is retentive of anything that comes to exist.

Q2 is what we would call Reality, material reality, physical. What separates it from Q1 is not so much a different ontology but a different perspective granted by extra dimensionality, and with that extra perspective comes boosted space. Space existed in Q1, as Euclidean volume. But Q2 is relative space, and that confers a whole range of different behaviour.

Q3 is also a relative space, but with an internal perspective, and on consideration I reckon Q3 is best labelled Internality. But it is an internality that looks out on boosted reality; it's an inner space that views outer space; it's a perspective that is holographic, i.e. it thrives only on what exists at its boundary.

Q4 is the space you are left with once the perspective on Reality is removed. With the boundary no longer a consideration, Q4 is whatever is left — the hologram without the machine, the ghost inside the machine, as Gilbert Ryle disparagingly called it, and Arthur Koestler reiterated in a more positive light.

If Tier 1 is about what things are, about the ontology of Existence in its various forms, Tier 2 is about the behaviour of Tier 1 — not what is, but what it does. So far I have assumed Tier 2 to equate entirely to Iconism, and while one view of Iconism may be that it belongs to Q3, it may yet turn out that the concept is applicable in all quadrants.

Appendix B — parked reference notes

Not draft, and not part of the argument. Retrieved background, kept because some of it names threads worth pulling. Each entry says what it is good for.

Nested triangles and simplices. The retrieved material behind Part I's heart section, and a lesson in checking retrieved material. What is correct in it: the medial triangle has sides half the length of its parents, parallel to them, with identical angles and a quarter of the area; the three corner triangles are congruent to it; the centroid of every nested triangle is the centroid of the original; nesting medial triangles for ever gives a total area of $4/3$ of the original.

What is wrong in it, and was corrected before use. It states that the sum of the perimeters diverges and then computes that same sum as exactly $2P$, two sentences apart; the computation is right and the statement is wrong. (The divergent case is the Sierpiński construction, where all three corner triangles are kept at every stage. That is a different figure.) And it gives the three-dimensional decomposition as four corner tetrahedra at half the volume between them, an inner midpoint shape at an eighth, and an octahedron at three-eighths — which sums to seven-eighths. The correct decomposition is four corner tetrahedra at an eighth each and one octahedron at a half. There is no separate inner midpoint shape, because in three dimensions the edge midpoints of a tetrahedron *are* the vertices of the octahedron. Its table of nested hypervolumes for $n \geq 3$ therefore rests on an object that does not exist.

Good for: the worked example at Part I, once corrected. Also as a standing caution. The error that mattered was not the arithmetic slip but the assumption that a two-dimensional construction scales — which is the very assumption this theory denies everywhere else, and which I very nearly imported into it because a retrieved source stated it confidently.

SU(2) and the double cover. SU(2) is the group of 2×2 unitary complex matrices with determinant 1. Written out, every element is a matrix in two complex numbers a, b with $|a|^2 + |b|^2 = 1$, which in four real coordinates is $x^2 + y^2 + z^2 + w^2 = 1$ — the unit 3-sphere. Its Lie algebra is three-dimensional, generated by the Pauli matrices. SU(2) double-covers SO(3), the group of ordinary 3D rotations: two SU(2) elements map to each rotation, a 360° turn multiplies the state by -1 , and 720° is needed to restore it.

Good for: Part V's double-cover section, which is the live use. If the base-loop composition is ever worked out explicitly, SU(2) and Spin(n) are the machinery to do it in — and voo7 suggests attempting it on the six mirror couples rather than the twelve individual reflections. Also worth noting that SU(2) acting on spinors is what tracks fermion spin, and that as a local gauge symmetry it governs the weak interaction — which is the force the Air-to-Water turn is assigned. Whether that is more than a coincidence is exactly the sort of thing that would want checking carefully before being claimed.

The Standard Model gauge group. $SU(3) \times SU(2) \times U(1)$. SU(3) has $3^2 - 1 = 8$ generators, giving eight gluons; SU(2) has 3, giving W^+, W^- and part of Z^0 ; U(1) has 1, giving the B boson. After electroweak symmetry breaking the count comes out as eight gluons, three weak bosons and one photon: **twelve gauge bosons**.

Good for: one curio and one caution. The curio is that the base loop has twelve extrinsic dimensions, and the Standard Model has twelve gauge bosons. Recorded, not banked — there is no argument connecting them and it would be easy to talk oneself into one. (voo7 adds a second-order curio and treats it the same way: those twelve extrinsics pair into six, and the bosons group into eight, three and one, which do not. If anything, the mirror structure makes the coincidence less tempting, which is a point in its favour.) The caution is that the four forces in the loop's table are assigned to the four *turns*, the intrinsic dimensions, not to the twelve extrinsics, so any attempt to line the twelve up with the bosons would be pushing against the architecture rather than with it.

Garrett Lisi's E8. In 2007 Lisi proposed unifying the Standard Model with gravity by mapping every fundamental particle to a root vector of the 248-dimensional exceptional Lie group E8, with twenty empty slots read as predicted particles. The reception was hostile and the technical objections stuck: Coleman-Mandula forbids mixing spacetime and internal symmetries into one group; Distler and Garibaldi showed E8 cannot accommodate the chiral structure of the Standard Model fermions; and the three generations do not fit without altering the theory.

Good for: a cautionary case rather than a resource. It is the nearest recent attempt at exactly the kind of thing this account could be mistaken for — geometric unification by finding the right structure — and it failed on chirality and on the generations. Both are questions this account will eventually be asked. Better to know that in advance than to be told it by a referee.

The elementary transforms. Translation ($x \mapsto x + v$); scaling ($x \mapsto cx$); projection, the idempotent maps $P^2 = P$ onto a lower-dimensional subspace. Composing two reflections in

distinct hyperplanes gives a rotation through twice the angle between their normals — the Cartan-Dieudonné theorem.

Good for: Cartan-Dieudonné is in Part II, where it does work, and is what makes the six mirror couples of Part III worth trying in the double-cover calculation. The others are listed because the Mandala uses none of them, and it is worth being able to say so: there is no scaling in the architecture, no translation, and no projection. Everything is rotation, reflection, or involution. If that turns out to be a limitation rather than a discipline, this is where to look first.

End of working draft v007.